

# Time Series Analysis For Troubleshooting In Paper-making Operations

Arun S. Mujumdar

## NOTATION

d	distance
E (f)	Frequency spectrum function
f	frequency
F(u)	cumulative probability distribution function
I	intensity of fluctuation
N	number of discrete data points
P(u)	Probability density function
R(t)	autocorrelation function
t	time
T	integration (or averaging) time
u(t), w(t)	time-varying quantities
V	velocity
Subscript	
rms	root meansquare value
overbar	indicates time (temporal) average
Prime	indicates a root-mean-square value

Arun S. Mujumdar

Department of Chemical Engineering  
McGill University Montreal,  
Quebec Canada

*With the continuing trend towards higher speed and greater efficiency operations in paper manufacture it has become essential to diagnose and correct problems arising from physical property variations in time and space. Since such variations are generally periodic and/or random and occur over a range of frequencies it is recommended that serious attention be given to using well established techniques of time series analysis. Although these powerful techniques have been applied successfully for several decades in numerous disciplines the paper engineer (technologist) has not generally been conversant with these techniques and the interpretation of results thus obtained. It is the objective of this paper to provide an introduction to the subject and outline a number of possible applications in the paper industry.*

## Introduction

It is generally recognized that the paper industry has traditionally been very conservative all over the world so that changes in existing machinery and use of more efficient novel techniques of pulp and paper making are not accepted as readily as they should be. This "inertia" may be explained partially in the light of the long life span of a typical paper machine and the high capital cost. However, it is more difficult to explain the fact that the industry, with notable exceptions to prove the rule, has also lagged decades behind most other manufacturing industries in utilizing well-established and relatively less expensive diagnostic tool which would certainly lead to faster more accurate trouble-shooting and/or more efficient designs. The

techniques of time series analysis is a case in point. The financial incentive for increased efficiency can hardly be over emphasized. Jones<sup>1</sup> has discussed the financial implications of a better machine operation in the Canadian industry. The figures would also apply at least semi-quantitatively to other parts of the world.

The objective of this paper is to outline the basic techniques of time-series analysis i. e. statistical representation and interpretation of time-varying data and to discuss briefly possible applications in the paper industry. To the author's knowledge no such attempt has been made in the trade literature.

It is well known that numerous design imperfections or changes in the operating variables lead to

undesirable property variations in the machine (MD) or cross-machine direction (CMD) in a typical paper machine. For example, an improper design of the headbox causing large regions of recirculation or periodic shedding of strong vortices, or a maladjustment of the slice can give rise to unacceptable basis weight variations in both MD and CMD. These in turn can accentuate "blowing" in the press section or "flutter" in the first dryer section and eventually to wrinkles that are totally unacceptable in the modern press-room. Basis weight variations may also arise due to flow oscillations caused by a wrongly-placed valve, a poor design of the approach piping for stock flow or a bad distributor. Furthermore, vibration of the press section due to passage of the felt seam, roll untrueness and ovalness or the basis weight variation itself could conceivably contribute to this basic problem of papermaking. While the above list is by no means extensive it does illustrate the fundamental justification for use of the techniques of time-series analysis: that basis weight variations in MD and CMD can occur due to a wide variety of causes some of which may be coupled (inter-linked).

Using a suitable on-line sensor for basis weight or moisture at one fixed location on the machine one would typically obtain a variation with time as shown in Figure 1. The signal is visually seen to be random in frequency

and amplitude. Obviously no useful information can be obtained from such a trace without further conditioning.

Band-pass filtering of the signal i. e. electronically extracting parts of the signal lying between two closely spaced frequencies one may locate a number of prominent frequencies buried in the "noise". An extension of this essential concept leads to a power spectrum (to be defined and explained in a later section) of the signal which gives valuable information about both the frequency of occurrence and magnitude of the variation. By application of this technique to study the power spectra of vibration of the press roll, pressure or velocity fluctuations in the approach piping or the headbox one can generally locate the problem areas and take suitable corrective steps either in operation or in the dryer.

Another important application may be found in a detailed study of sheet flutter which has been ascribed to a number of possible mechanical and aerodynamic causes<sup>2,3,4</sup>. This problem will be dealt with in some detail in a later section.

Mechanical vibration of the disks is known to have deleterious effect on both the refiner pulp and the life span of the disks. This is another area of crucial interest in refiner groundwood operation. Indeed, whenever rotating elements are present (and these abound in a paper making operation) it is always

wise to examine the spectral characteristics of the vibration and noise emanating from such devices. These can lead to both mechanical and process problems that may be impossible to correct fully.

The techniques of time series analysis have been used with great success in numerous disciplines ranging from aerospace engineering to ocean technology. Examination of vibration problems in rockets and oscillation-caused rupture of underwater cables are just two applications from these two disciplines. Researches in practically all physical and life sciences have been since the time series analysis techniques to explain phenomena that could not be explained in any other way. Langenthal<sup>5</sup> and Mujumdar<sup>6</sup> have listed numerous references to illustrate the breadth and depth of such techniques. No attempt will be made even to mention these applications; the interested reader is referred to these two texts for details. Mujumdar and Douglas (6 through 11) have used techniques to study periodic eddy-shedding from cylinders and spheres in turbulent flows. Based on the author's experience with applications in turbulent flow following is a simplified discussion of the various terms used to represent statistically time-varying data; how these are measured, and what they mean physically.

#### Definitions of Terms Commonly Used

Let  $u(t)$  be a continuous random

function of time (t) in the following discussion it will be assumed to be statistically stationary i. e.

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T u(t) dt = \bar{u} \text{ (independent of time).} \quad (1)$$

More complicated techniques of measurement and analysis are required for non-stationary stochastic (statistical) variables. However, these are encountered less frequently. References 5 and 6 give detailed accounts of these methods.

Let

$$u(t) = \bar{u} + u'(t) \quad (2)$$

(total) (mean) (fluctuating)  
which, because of Equation (1), means

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T u'(t) dt = 0$$

i. e. the long-time average of the fluctuating component is zero. Because of stationarity the dc or mean component of u(t) need not be considered in further discussion.

Root-Mean-Square Value,  $U_{rms}$

The simplest and most commonly measured property of a time-varying signal is its root-mean-square (rms) value i. e.

$$u_{rms} = \left( \sum_{i=1}^N u'^2 \right)^{1/2} \quad (3)$$

Generally it is more meaningful to consider the relative magnitudes of  $u_{rms}$  and  $\bar{u}$ . Hence, we define

$$I = \frac{u_{rms}}{\bar{u}} \quad (4)$$

Probability Density Function, P(u)

The amplitude probability density function, P(u), provides the simplest statistical property of a random variable in the amplitude domain. Referring to Figure 1 P(u) is defined such that P(u)du gives the probability of finding a value of u between  $u - \frac{\Delta u}{2}$  and  $u + \frac{\Delta u}{2}$ . Expressed mathematically,

$$P(u) = \lim_{\Delta u \rightarrow 0} \frac{\Pr \left\{ u - \frac{\Delta u}{2} \leq u \leq u + \frac{\Delta u}{2} \right\}}{\Delta u} \quad (5)$$

Often an integral or cumulative probability distribution function F(u) may be used where

$$F(u) = \int_{-\infty}^u P(u) du \quad (6)$$

Figure 2 illustrates the various types of density and distribution functions commonly encountered. Noteworthy is the shape of P(u) when the signal consists of a sinusoidal wave superimposed on a random signal. The sharp peaks in P(u) represent the amplitude of the periodic waveform.

It is obvious that if the mean or dc component is filtered out prior to the analysis the PDF's would not contain information about the mean value. The only difference between P(u) and P(u'), for example, is the shift of the origin along the amplitude abscissa. Furthermore it may be noted that P(u) and F(u) are continu-

ous functions except when u(t) is a binary signal or a periodic pulse train. Further discussion of the latter types of signals is not included as these types of signals are not likely to appear in the conventional papermaking operations.

Correlation Functions

The correlation function gives a statistical representation of the degree of similarity of the random signal with itself (or another signal obtained simultaneously) in the time domain. The autocor-

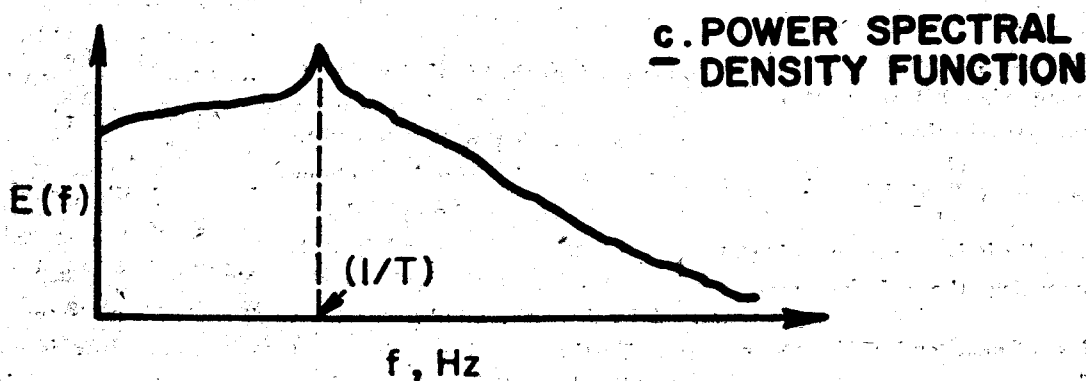
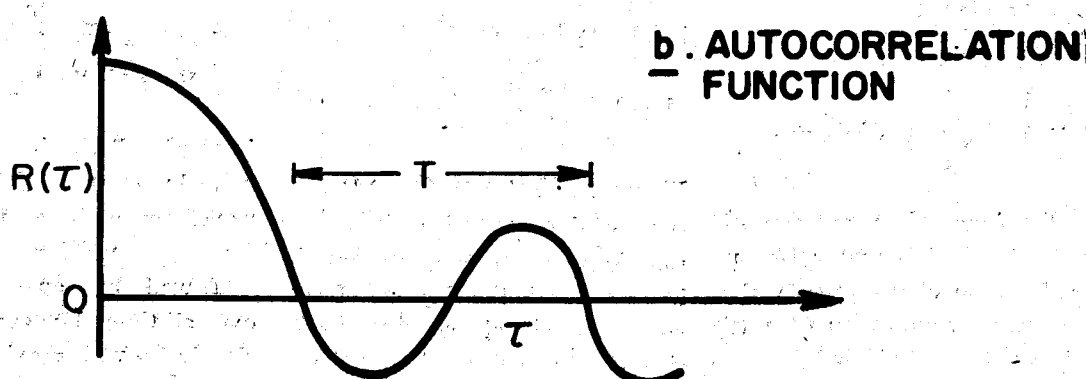
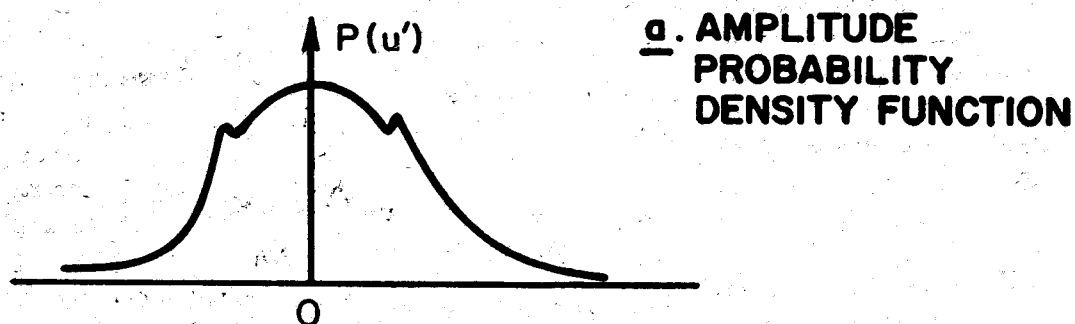
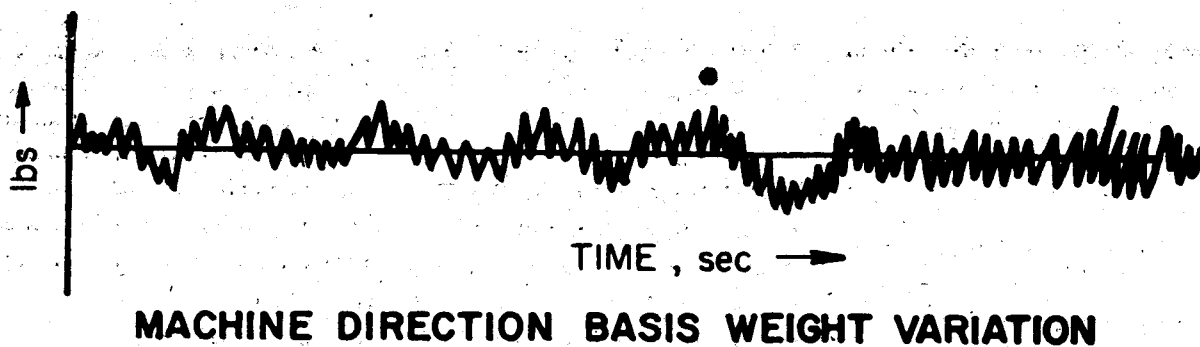
relation function, R(τ) is defined as

$$R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^T u(t) u(t+\tau) dt \quad (7)$$

In words, R(τ) is obtained by point-by-point multiplication of a waveform with a time-shifted (by τ) version of itself, followed by an integration over all time. For any waveform, clearly R(τ) is maximal at τ=0 (since degree of similarity between identical signals is perfect). For a random signal since there is equal probability of u being positive or negative about the mean ( $\bar{u}$ ) R(τ) tends to level off at  $u_{rms}^2$ .

Figure 3 illustrates the various types of correlation functions commonly encountered.

The similarity between two different waveforms is u(t) and w(t) obtained by considering the



**Fig. 1: Basis weight variation with time**

cross-correlation function  $R_{uw}(\tau)$  defined as

$$R_{uw}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T u(t)w(t+\tau)dt$$

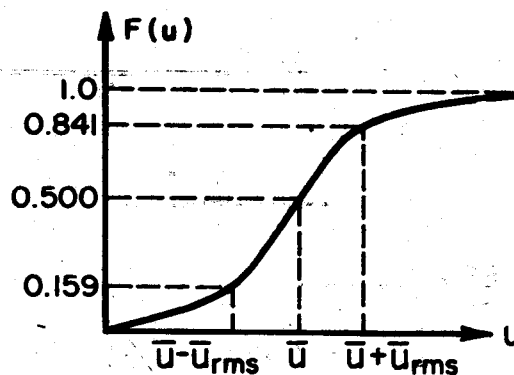
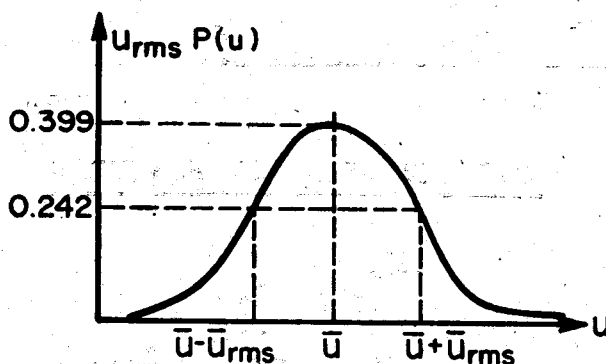
or

$$R_{vw}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T w(t)u(t+\tau)dt$$

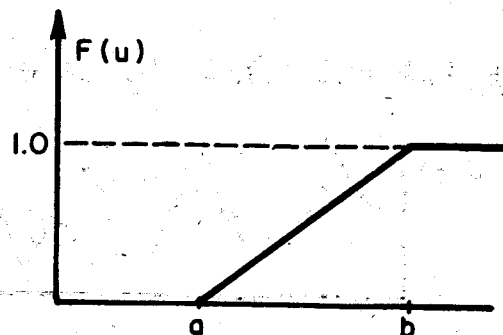
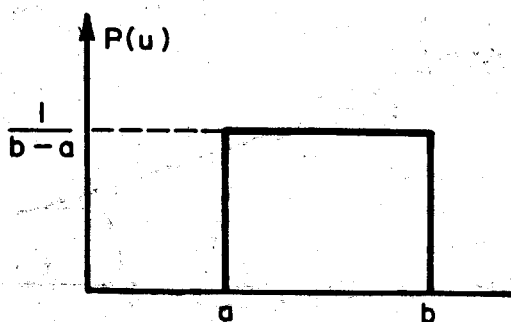
The cross correlation function does not display symmetry about  $\tau=0$ . However, it does not display symmetry about the ordinate when  $u$  and  $w$  are interchanged. It is important to note the following additional properties of  $R_{uw}(\tau)$  and  $R_{wu}(\tau)$ .  
 $|R_{uv}(\tau)|^2 < R_u(0)R_w(0)$

$|R_{uv}(\tau)| < \frac{1}{2} [R_u(0) + R_w(0)]$  i. e. square of its magnitude is never greater than the product of the power contained in the two signals and that its magnitude is also never greater than the average of the power contained in the two signals.  
 Cross-correlation analysis has

### a. GAUSSIAN RANDOM VARIABLE



### b. UNIFORMLY DISTRIBUTED RANDOM VARIABLE



### c. SINE WAVE BURIED IN RANDOM NOISE

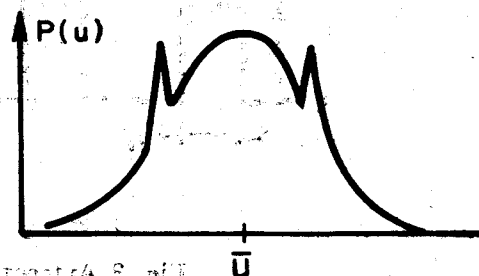
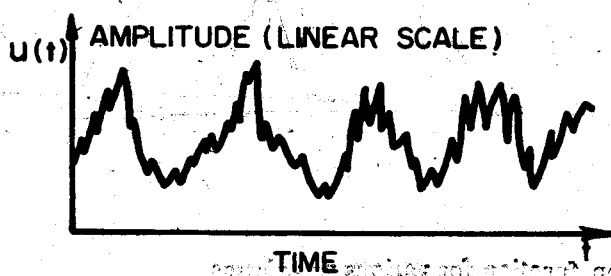
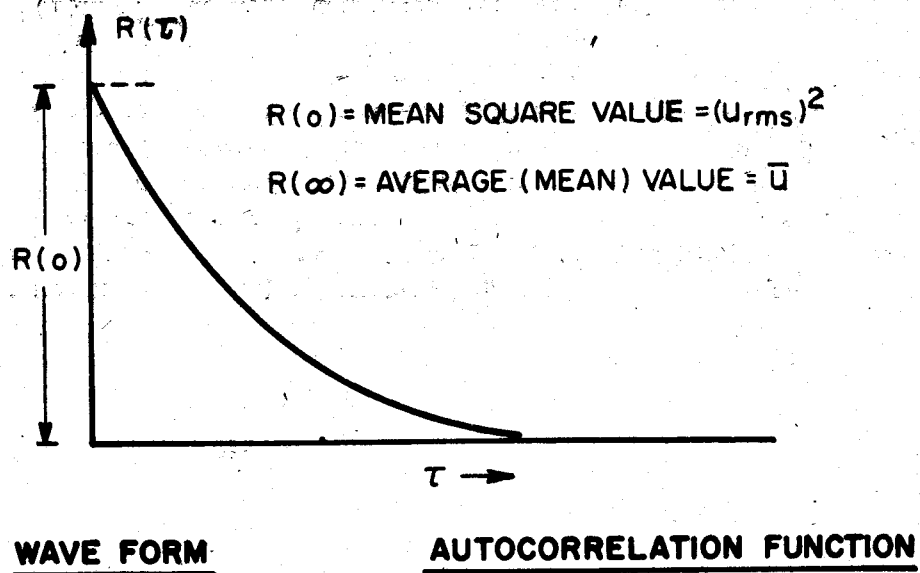


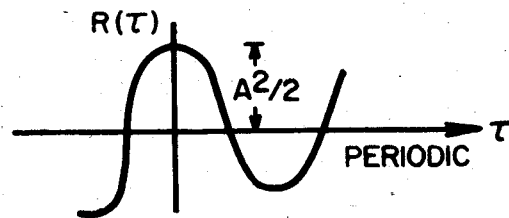
Fig. 2 Examples of probability density and distribution functions



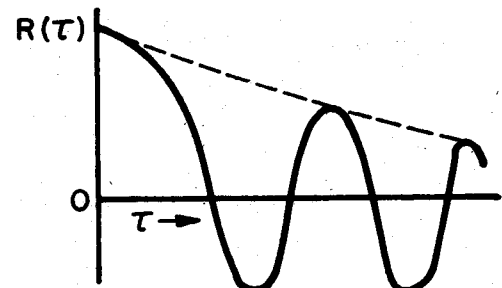
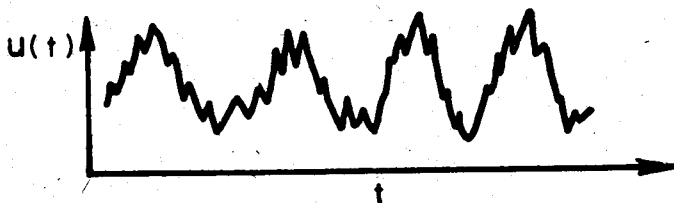
**a. SINE WAVE**

$$U(t) = A \sin(\omega t + \theta)$$

$$R(\tau) = \frac{A^2}{2} \cos \omega \tau$$



**b. PERIODIC SIGNAL BURIED IN RANDOM NOISE**



**c. PERIODIC PULSE TRAIN**

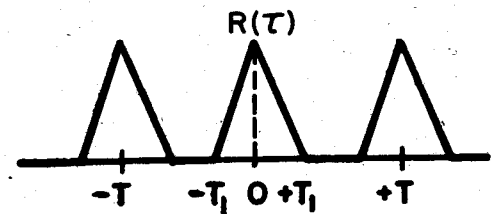
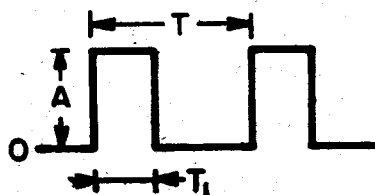


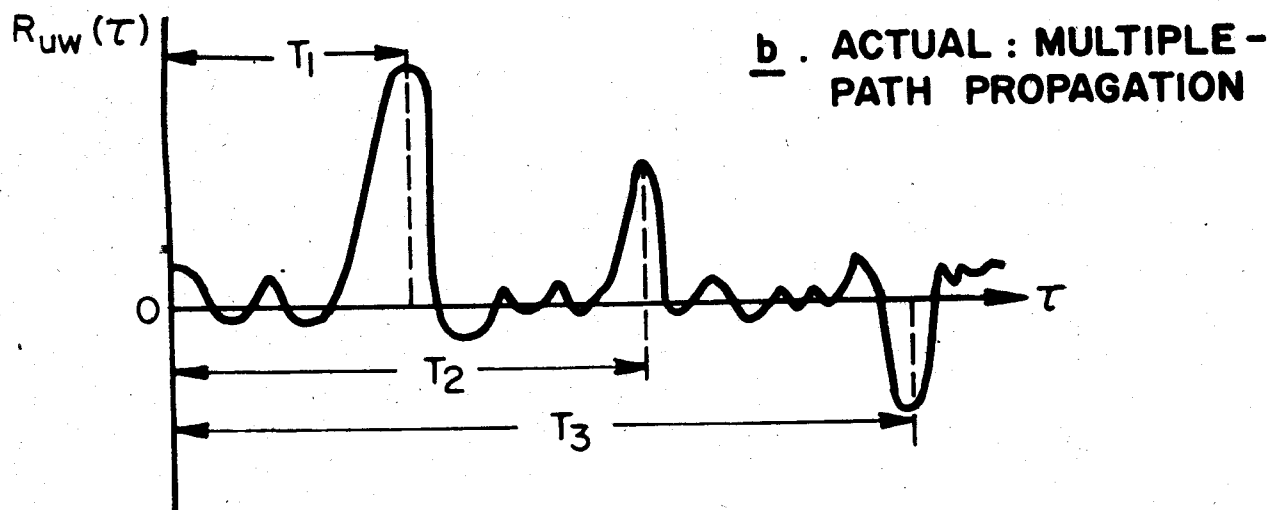
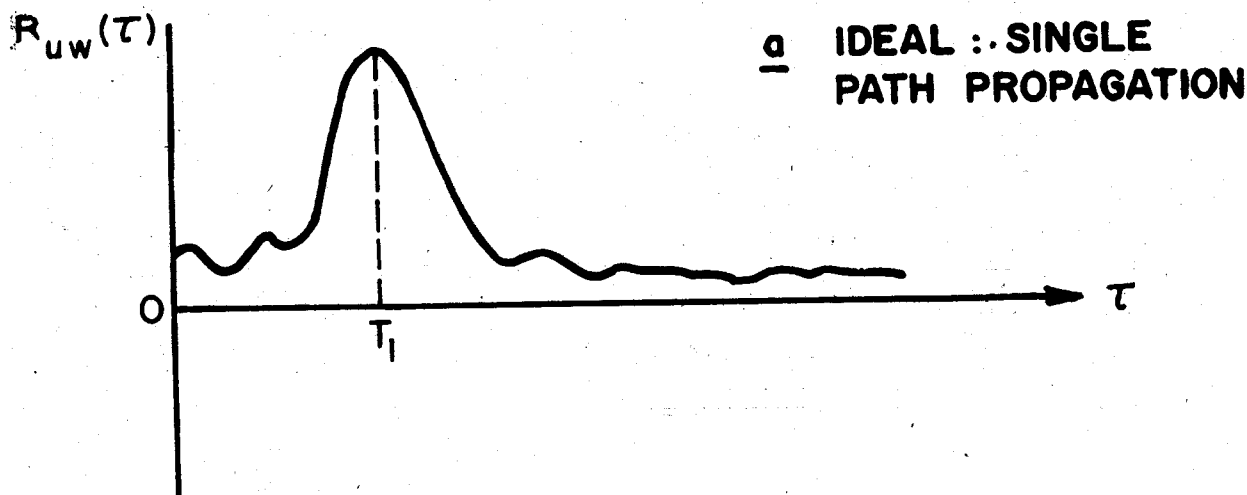
Fig. 3. Autocorrelation function for various waveforms

found numerous applications in industry although none are reported (to the best of the author's knowledge) in the paper industry journals. Perhaps the most common applications which are also relevant to the industry are in the area of acoustics and system response identification (or transfer function determination).

In a paper mill, there are numerous sources of noise. In is desired to determine the degree of relative effect of the various sources of the noise level at an arbitrary location in the mill. Since it is impossible to isolate the noise generators the technique of cross correlation may be used. A cross correlation function of

the type shown in Figure 4 may be obtained by cross correlating the sound levels created by each of the noise sources, one at a time, with the sound level at the point of interest.

If sound propagated in one direction the cross correlation function would be of the form shown in Figure 4(a), consisting of one



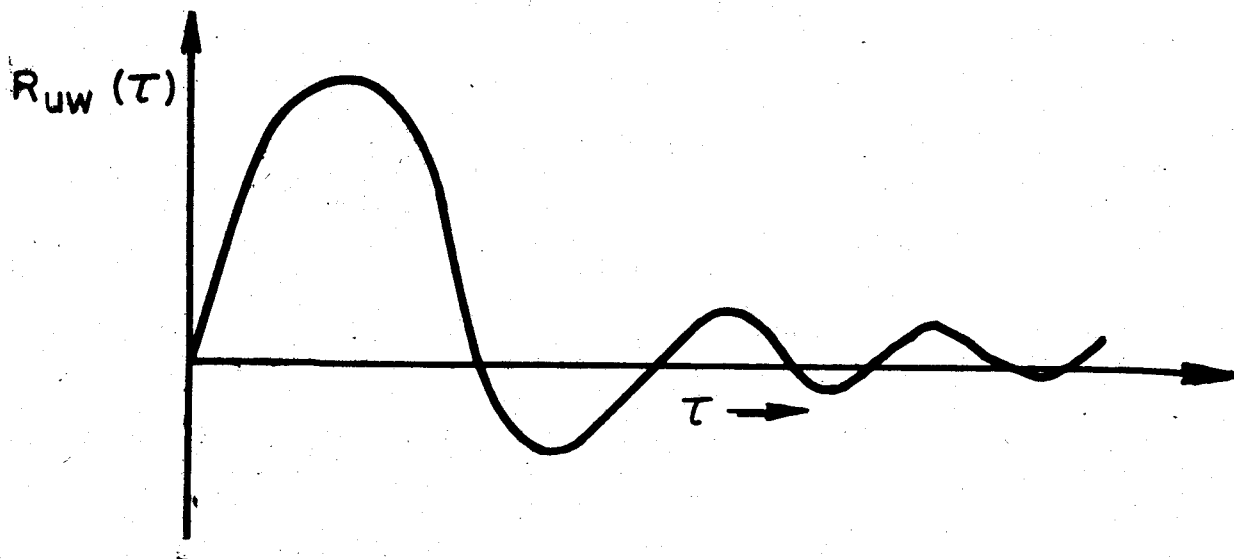
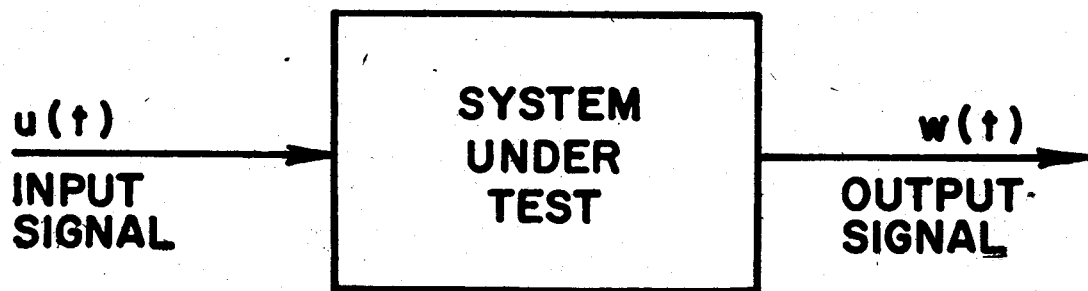
g. 4 Cross-correlation functions of sound levels a. Ideal b. Actual

peak at a time delay equal to the propagation time. Actually the sound is transmitted by various modes—reflections off walls and other pieces of equipment, etc. The various peaks ( $T_1, T_2, T_3$ ) give the propagation times for

each path. (figure 4b). Thus it is possible to locate the design information needed for noise control. This procedure is used commonly in problems of multipath determination encountered in vibration analysis, radar, so-

nar and communication problems. It is often necessary to determine the impulse response of a physical system. Use of impulse or sine wave testing lead to considerable difficulties in practice, one of the major ones being

## **SYSTEM RESPONSE IDENTIFICATION**



## **CROSS-CORRELATION FUNCTION**

Fig. 5 Set-up for System Response Identification



the generation of the input signal itself. Valuable information concerning the delay through the system (time to first peak) overshoots and oscillations is obtained by cross-connection of the input and output waveforms. This particular application is particularly useful in papermaking from the process control point of view.

In general the correlation analysis provides time or time delay information. Since time ( $t$ ), velocity ( $v$ ) and distance ( $d$ ) are related through

$$t=d/v$$

knowledge of  $t$  (time of occurrence of a peak) and either  $v$  or  $d$  allows determination of the other parameter. Thus, in a hot-rolling process the speed of the moving hot sheet can be measured and controlled by cross correlating the output signal from two photocells separated by a known distance. Essentially the same technique has been used to measure velocity of blood in capillaries and determination of scales of turbulence. Although expensive, in some instances, this happens to be the only reliable method of measurement of velocity.

It may be noted parenthetically that both the probability and correlation analyses involve averaging or integration process and hence are inherently noise-attenuating processes. Although signal enhancement is one of the major applications of these analyses this application is not considered further as it is unlikely to be found in papermaking practice.

### Spectrum Function, $E(f)$

The power spectral distribution function is simply a frequency-domain representation of the correlation function; the two are Fourier transforms of each other. Without going into the details of the mathematics one may define  $E(f)$  of a time-varying signal as a measure of the power (or mean square amplitude) distribution as a function of frequency,  $f$ . Thus,  $E(f) df$  represents the power contained in fluctuations in the frequency bandwidth  $f - f/2$  and  $f + f/2$ . Clearly if the waveform is a pure sine wave then  $E(f)$  would be a delta function centered at the frequency of the sine wave. Figure 6 shows a typical power spectrum function for a signal consisting of a sine wave (and its harmonics) and random noise. Several excellent references treat the subjects of measurement and interpretation of power spectra from both the theory and applications point of view. Interested readers are referred particularly to References 12 through 16.

In the absence of published data Figure 7 gives a rough estimate of the frequency bandwidths that may be encountered in a paper-machine. The bandwidths will certainly depend on the design and operating speed of the machine. It is important to note that in papermaking we are concerned primarily with very low frequency variations. Although this simplifies the frequency response requirements of the transducer it makes it difficult to use ordinary audio or vibration

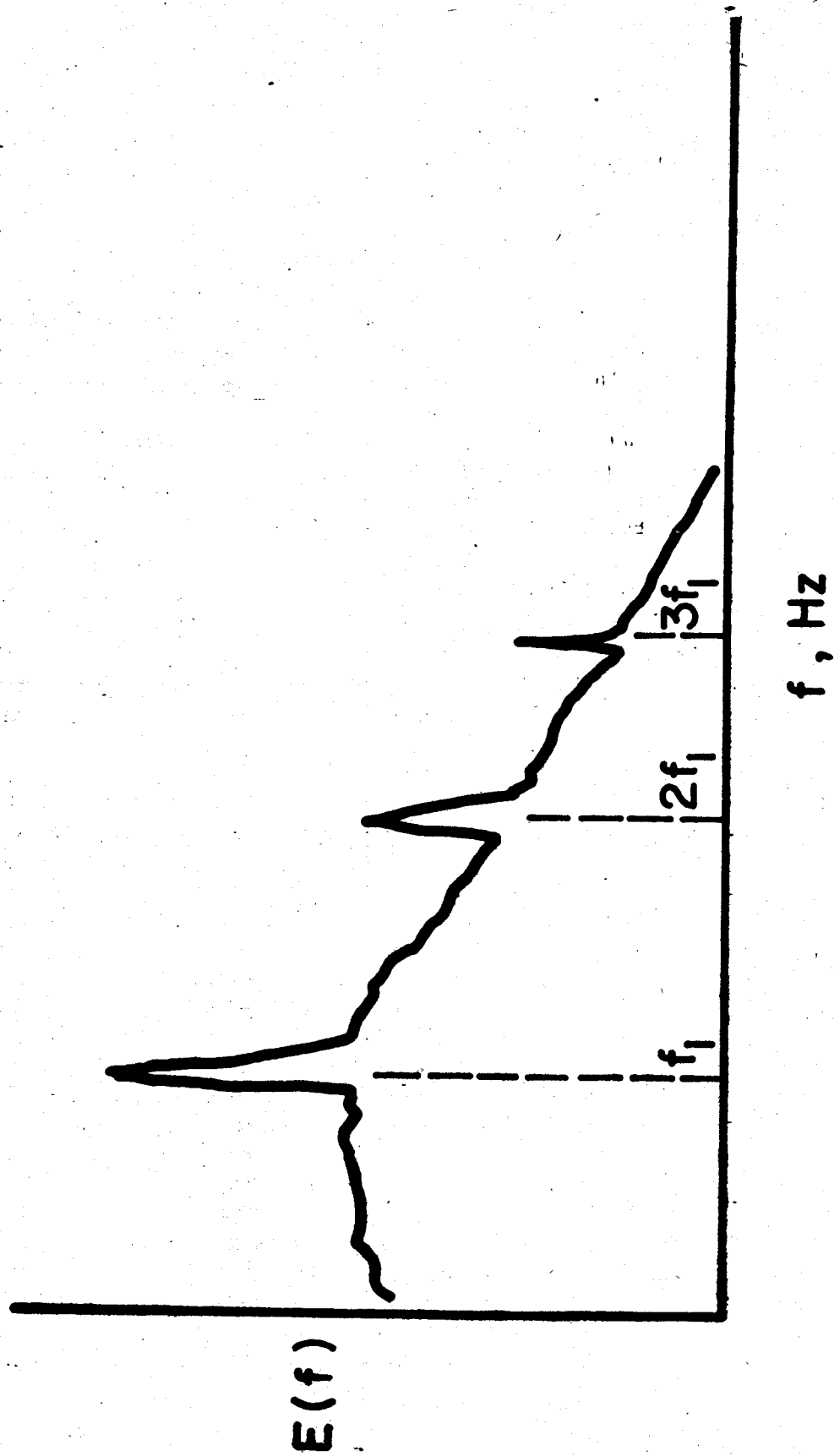
analysis instrumentation without frequency transformation.

### Cross-Power Spectrum

This is a Fourier-transform of the cross-correlation function and hence is a frequency-domain measure of the degree of similarity between two signals at each frequency. In actual practice the cross-spectrum is an intermediate step leading to the cross-correlation function, the transfer function and the coherence function (which is a measure of the similarity between two signals at each frequency). It may be noted that the cross-power spectrum is complex; its real part is called the co-spectrum and the imaginary part the quad-spectrum. Unlike the power spectral density function, this function contains phase information.

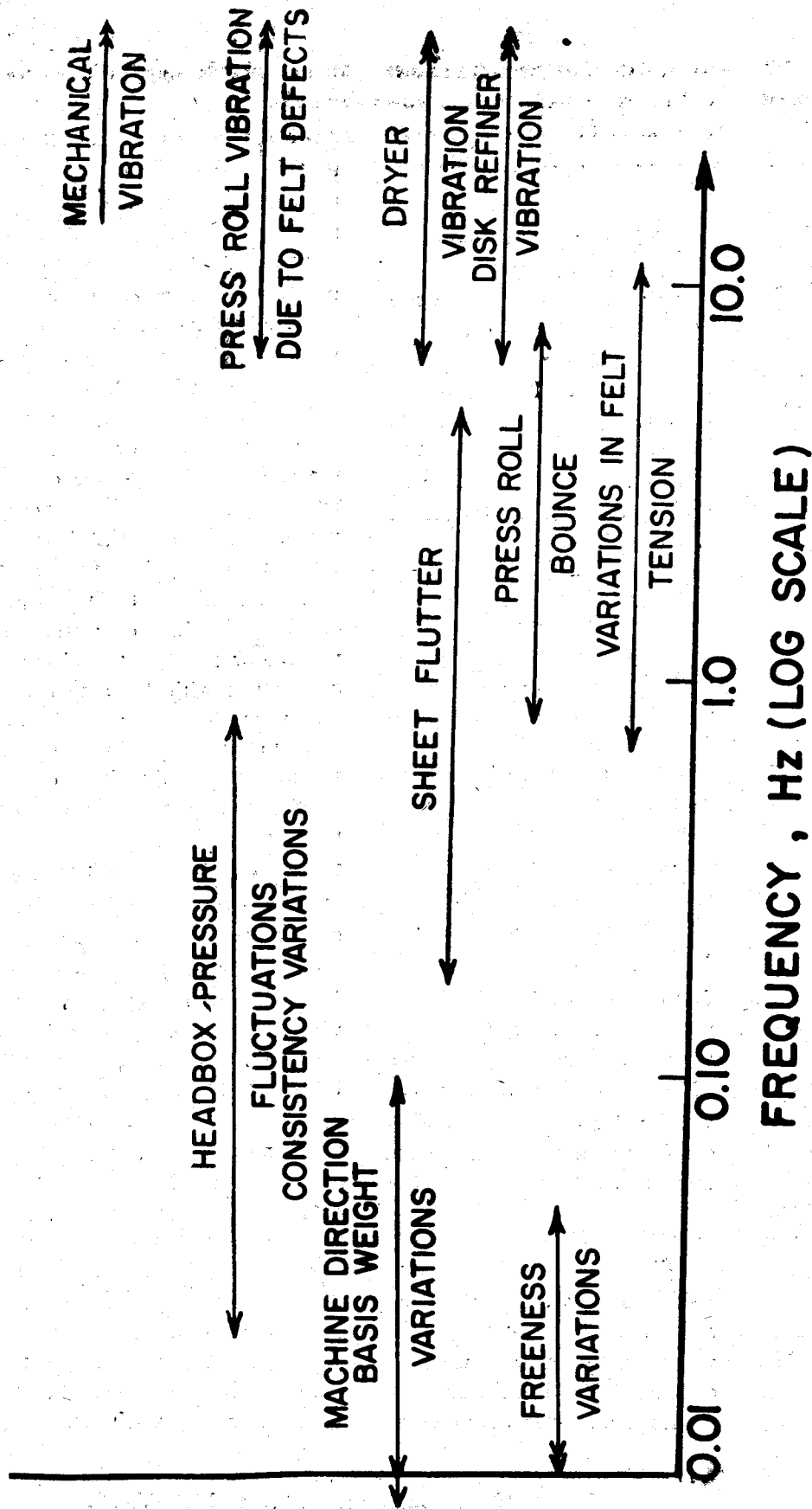
### Measurement Techniques

The amplitude, time and frequency domain statistical parameters can be obtained using analog, digital or hybrid techniques of data acquisition and processing. Numerous commercial packages are available in the market; generally the price tag varies significantly depending upon the accuracy, speed of analysis, frequency bandwidth, storage and output capabilities etc. Hybrid techniques involving analog-to-digital conversion prior to actual (digital) computation and final conversion to a analog output signal appear to have become popular. In any event, digital techniques are more versatile and very flexible; their chief disadvantages



## POWER SPECTRUM OF RANDOM NOISE WITH SINE WAVE AND HARMONICS

Fig. 6. Spectral distribution for a random signal containing a sinusoidal wave with harmonics



**NOTE**  $\longleftrightarrow$  IMPLIES UPPER OR LOWER LIMIT EXTENDS BEYOND SCALE SHOWN

$\longleftrightarrow$  **FREQUENCY BANDWIDTH OF INTEREST**

Fig 7 A schematic of the possible bandwidths of various disturbances in a paper machine.

being the requirement of fast A-to-D conversion equipment and access to a fairly large digital computer. The A-to-D conversion must be achieved at a rate at least twice the highest frequency expected in the signal being processed (but preferably three times). On-line evaluation may be impossible in the field. This problem may be obviated by recording the analog signal on a magnetic tape (either in analog form or in digital form if it is to be processed on a digital computer) for later analysis. Magnetic tape recording offers a further and often critical advantage of "frequency — transformation". Most tape recorders have multi-speed capability. By recording the signal at one speed and playing it back at another (to suit input requirements of the analyzer available) the frequency bandwidth may be varied to improve accuracy and/or reduce the measurement time appreciably.

If the signal contains very low frequency components (near dc) which would invariably be the case in a papermachine wet end because of its large "inertia", the magnetic recorder must operate in the FM (frequency modulation) or PD (pulse-duration) mode so as to record such low frequencies. Since most reasonably-priced spectrum of wave analyzers do not cover such low frequencies it may be desirable to frequency-transform the signal by playing back the recorded signal at up to 100 times the "real time". Of course, this pro-

cess by itself may introduce noise due to cross-talk, flutter and wow and sufficient care must be exercised in a frequency transformation.

Finally, because of the popularity of the digital techniques of statistical data processing a number of software packages are available in the market and a number of computer programs are found in published literature<sup>20</sup>. The numerical aspects of computation of the correlation functions and spectra are dealt with by Ralston and Wilf<sup>21</sup>.

A big impetus to the digital processing techniques has been given by the development of the fast Fourier Transform (FFT) algorithm. If  $N$  is the number of discrete data points to be used for statistical analysis, with FFT the computational effort increases only as  $N \log N$  and not  $N^2$  which is true for the older algorithm. References 22 through 25 cover the theory and applications of FFT.

For the range of applications in papermaking as envisioned by the author analog system with an FM magnetic tape recorder would yield valuable information concerning the time-varying phenomena to be studied.

#### **Suggested Applications in Papermaking**

Aside from general process applications (such as relationships between various process variables, control loops, identification of malfunctioning parts etc.) a num-

ber of possible applications may be cited.

At the wet end the instability of the feed system is generally a problem area. Variations in freeness, consistency etc. lead to basis weight variations which in turn cause expensive downtimes. Cross-correlation between these variables monitored on-line will provide valuable information about the problem and suggest ways of alleviating or eliminating the problem. Vortex-shedding is another hydrodynamic phenomenon which is undesirable as it leads to fluctuating flow rates and fluctuations in the stock consistency. Use of a suitable velocity or pressure transducer in conjunction with the auto correlation or spectrum analysis will lead to detection of this vexing phenomenon. Suitable design changes can then be effected and tested similarly.

The flow rate of the thick stock may perhaps be monitored by a cross-correlation technique.

Recently Antos, Patricelli and Whiteside (26) have discussed a variety of vibration problems encountered in the press rolls and higher nip loadings demanded by higher machine speeds. Variations in air permeability, mass, thickness and/or tension in the felt can cause severe roll bounce and press vibration. Rather complex waveforms are generated by an accelerometer monitoring the machine vibration (see Figure 4 Reference 26). Although the authors did extract useful information from a visual study of

such traces additional information could be obtained by resorting the data to correlation and/or spectrum analysis. For example, since the vibration emanates from a variety of sources the relative importance of each could be determined through a systematic statistical analysis.

Another potentially significant area particularly in need of such study is the analysis of the sheet flutter limiting the top speed of most modern paper machines. The wavy motion of the sheet could be due to air flow oscillations, felt passage frequency, press and dryer vibration, ovalness of the rolls, nonuniformity in basis weight, air permeability and felt thickness etc. Using a photographic technique or a suitable optical or mechanical displacement transducer which gives a linear voltage output corresponding to normal displacement of the wet web, one can cross-correlate it with simultaneous variations in a number of possible parameters influencing the phenomenon. In the absence of such information ideas regarding the relative importance of the various causes remains purely speculative. For the same reason any design modifications attempted to rectify the problem are also of the trial-and-error type and generally ineffective. Perhaps the single most important parameter of interest in the sheet flutter study is the magnitude of the most preferred wavelength and how it is related to the web tension, length of the open draw and the machine

speed.

Finally, another important problem that can be handled readily by spectral techniques is the problem of instability of the mechanical disk refiner operating at high consistency. Here the "steam blowback" is a hydrodynamic problem while vibration of the disks is a mechanical one. Both are very important and deserve a careful experimental investigation.

#### Closure

Since papermaking is essentially a time-dependent phenomenon numerous additional application can be suggested. Indeed the only limitation perhaps is the imagination and ingenuity of the paper engineer confronted with design or operational problems. There is little doubt that such techniques will be essential and will eventually become commonplace with development of higher speed machines that will not permit guesswork or trial-and-error diagnostics.

#### Acknowledgement

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