Fundamentals of Control Charts

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The statistical quality control methods and control charts based on them were developed by Dr.W.A. Schewart in the Bell Telephone Co. of America, as early as 1924. The SQC methods made good stride during World War II in order to meet mass production and stringent requirements for accuracy in meeting exact specifications. Control charts help to set up quality standards which management strive to achieve, help in attaining standards and finally reveal to what extent the goal is achieved. Thus they are instruments for use in specification, inspections and production. In mass production same product is made continuously but no two products are identical in all respects. This difference between the products made on same machine and conditions is due to inter-action of various causes. These causes can be controlled to some extent to produce articles with greater similarity, but can never be eliminated. The basis of control charts is the differentiation of causes of variation in quality of any product. There will be assignable as well as unassignable causes. The unassignable causes are aggregate of many small causes attributing to small variation but none plays a major role in creating the variation. These complex causes are known as chance variations. The SQC techniques help to separate these assignable causes of variation. Many production troubles are diagnosed and corrected with the help of SQC. By identifying certain quality variation as inevitable chance variation, the SQC methods tell us when to leave the process alone and thus prevent unnecessary and frequent adjustment that tend to increase rather than decrease the trouble.

Basic Concepts of Statistics :

The quality characteristics for any product can be of two types i.e. qualitative and quantitative. A non-measurable characteristic of any product is called an 'attribute' such as quality of an article as 'good' or 'bad'. The measurable quantitative characteristic is known as 'variable' or 'variate' which varies from one individual to another. It is denoted by x. Strength properties of any paper are its 'variable' quality characteristics. When a variable assumes any value within a range, it is known as continuous variable such as substance of paper. It is termed as discontinuous when it cannot assume fractional values for example number of pin holes in a sheet of paper or waxpick number of paper. A set of huge data is condensed by methods of classification of ranking before it becomes intangible. The variables may be put in different intervals known as class intervals and frequency of occurrence in each class is noted. The way in which variables are distributed in class intervals, is termed as 'frequency distribution'. In order to distinguish between any two distributions, it js necessary to know their averages (the variable around which distributions are centred) and dispersion (the extent to which variables are spreading). The arithmetic mean or average of a series is the quotient of sum of all values by their total number.

$$\text{Average} = \bar{X} = \frac{x_1 \div x_2 \div x_3 \div \dots\dots\dots\dots \div x_n}{N}$$

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The simplest measure of dispersion is the range (difference between maximum value recorded and minimum value recorded.

$$\text{Range} = R = x_{\text{maximum}} - x_{\text{minimum}}$$

Chief use of range is in SQC methods but it is not reliable for large size date. The most important measure of dispersion is Standard Deviation. S. D. denoted by '6', is the square root of the averages of the squares of deviation from mean.

$$\text{Standard Deviation} = \sigma = \sqrt{\frac{\Sigma\,(x_1 - \bar{x})^2}{N}}$$

A normal curve of error is good representation of chance variation. Normal curve pattern for an aggregate is necessary, but not sufficient condition for controlled production.

The control charts give evidence of control which is based on normal curve. The normal

$$\text{curve is } Y = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-m)^2}{2\sigma^2}}$$

where m = mean of distribution and

σ = Standard Deviation.

The total area covered between different limits of normal curve is :

$m \pm \sigma = 68.26\%$; $m \pm 2\sigma = 95.46\%$

$m \pm 3\sigma = 99.73\%$

Most of variations due to chance causes lie within $m \pm 3\sigma$ limits.

The Normal Curve

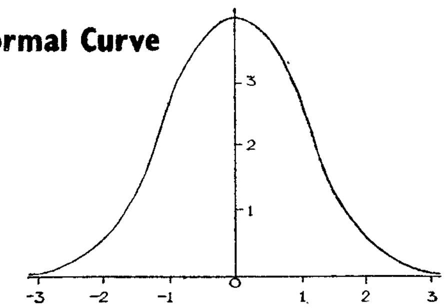


Fig. 1

Theoretical basis of Control Charts :

There are several ways of establishing tolerance or specification limits. They may be arbitrary figures based on experience or may be derived statistically. As long as production follows a chance variation, the normal law will apply to its product. Hence limits for such a controlled production will be based on the properties of normal curve. Whenever a sample is drawn from a production line, it serves not only for a spot check but indicates the trend of whole production. This 'whole' is termed as 'population' or 'universe' in statistician's language. The number of individuals in a sample or sub group is called its size (n). From samples drawn at intervals, their average ($\bar{x}$), range (R) or fraction of defectives (p) can be computed. These will be subjected to sampling fluctuations within a definite limit, depending upon the population. If on a chart, mean range or fraction defective for successive samples are plotted along with control limits computed from normal law, the result is a 'control chart'. Schewhart has illustrated that means or averages of random samples drawn from a universe are distributed normally, even in cases where distribution of individuals may depart from normality.

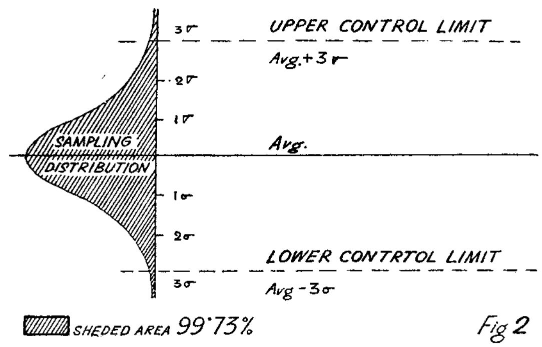


Fig 2

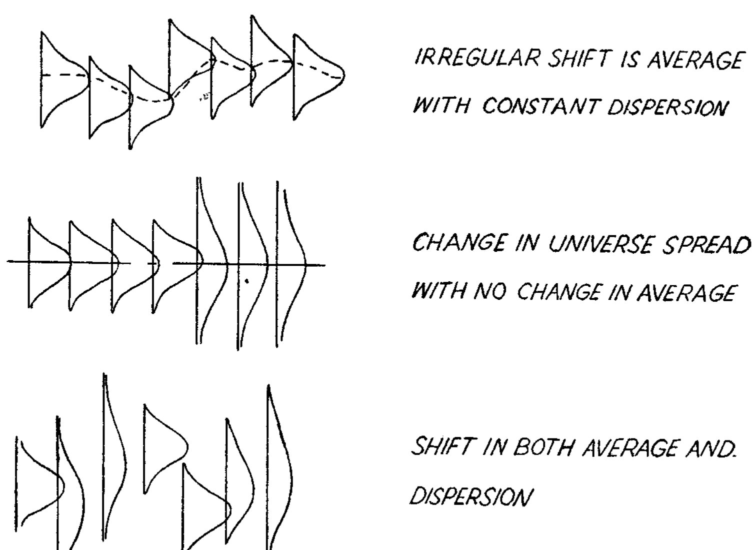
THEORETICAL BASIS OF CONTROL CHARTS

Control charts have an important role in detecting lack of control. The lack of control can be in three ways viz. in average, dispersion and both. The most common lack of control is shift in average. The position of different points also give a sufficient evidence of lack of control based on "Theory of Runs". There can be suspicion that the process average has shifted whenever :-

(1) 7 successive points lie on the same side of central line.

(2) 10 out of 11 successive points lie on the same side of central line.

(3) in 14 successive points at least 12 lie on the same side of the line.



Types of Lack of Control

Fig 3

Position of Control Limits and sub group size:

Setting up of control limits and sub group size is very much imperative for the effective working of Control Charts. Many a times it is possible to locate assignable causes from the control charts but it is tedious and uneconomic to remove them. Hence a set of economic and effective limits have got to be found out. Often the trouble is left uninvestigated as it is not revealed by the sample or else unnecessary hunt for trouble continues even though it is absent. Such troubles can be isolated by putting suitable control limits based on statistical theory. It has been found satisfactory to place control limits 3 S. D. (3σ) apart on either side of grand average. Sub group size should be such that assignable causes show their presence between samples and not among the members of the sub group. If the system is changing, then sample size should be as small as the average if samples do not mask the changes. A sub group size of 4 or 5 is most convenient. When control charts are made very sensitive to small shifts in process average, a sub group size of 10 or 15 is more effective.

The Variable Control Chart:

Control Charts based on measurable quality characteristics are termed as 'variable control charts'. The two variable control charts most commonly used are average or $\bar{x}$ - chart and range $\bar{R}$ - chart. $\bar{x}$ - chart is used to control the variation of process. R - chart is meant to control variability of process. The upper and lower control limits (UCL and LCL) on a $\bar{x}$ - chart are 3σ limits of sample average $\bar{x}$. The sample average is distributed with $\frac{\sigma}{\sqrt{n}}$ as standard deviation. (n is sampe size). In series of samples drawn from a population, S. D. can be calculated for each sample and then averaged. It involves lot of arithmetic. The population S. D. can be calculated from sample range also. Range of samples follows a continuous distribution and has a relation with population standard deviation.

$$\sigma = \frac{\bar{R}}{d_2} \quad \text{where } \bar{R} \text{ is average range}$$

and d_2 a constant factor depending upon sample size and derived from the distribution of range. As calculation of S. D. involves lot of arithmetic, the above method of estimation of S. D. from sample range and constant d_2 is used in practice.

Control Limits for $\bar{X}$ and $\bar{R}$ charts.

Limits	$\bar{X}$—Chart	R—Chart
Central Line	$\bar{\bar{X}}$	$\bar{R}$
Upper Control Limit (U. C. L.)	$\bar{\bar{X}}+A_2\bar{R}$	$\bar{R}\times D_4$
Lower Control Limit (L.C.L.)	$\bar{\bar{X}} - A_2\bar{R}$	$\bar{R} \times D_3$

Upper and lower control limits on both the $\bar{X}$ and R chart are 3σ apart from the Central line representing grand average. The constants A_2, D_3, D_4 and d_2 depend on sample size in any sub group. They have been calculated for different values of n (sample size) for rapid calculation. An extract of different values of above constants is given below. More details can be had from any Hand Book on Quality Control.

n	d_2	A_2	D_3	D_4
2	1.128	1.88	0	3.27
3	1.693	1.02	0	2.57
4	2.059	0.73	0	2.28
5	2.326	0.58	0	2.11

Example on Application of $\bar{X}$ & R charts :

It is a case study from bleaching section of a Pulp Mill. The quality considered is residual chlorine content lbs. per tonne of pulp after washing. The chlorine content has a significant effect in shade of paper. Abnormally high variations in average and range was noticed from control charts for a set of 20 sub groups each comprising of four two-hourly samples. The variations indicated on control charts led to the investigation of bleaching process. It was revealed that improper dosage of bleach liquor, abnormal variations in vat consistency and imperfect washing due to damaged screens were responsible for such erratic values of chlorine content.

The second part of control chart shows how the average and dispersion have been reduced by controlling various factors.

Table showing residual chlorine lb. per tonne of pulp before and after control.

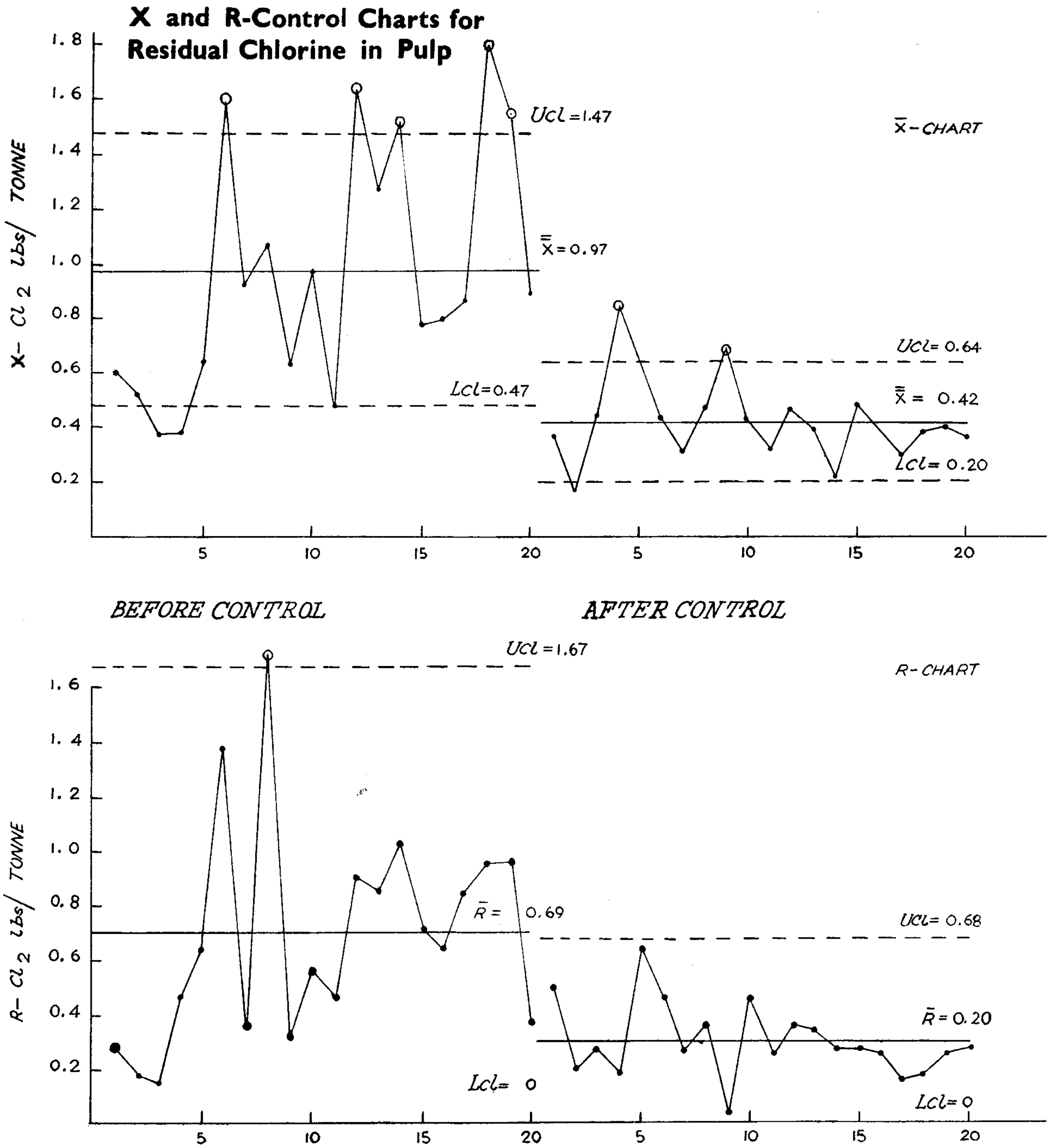
Sub group	Before Control		After Control	
	$\bar{X}$	R	$\bar{X}$	R
1	0.60	0.28	0.36	0.50
2	0.52	0.18	0.16	0.18
3	0.38	0.14	0.44	0.28
4	0.38	0.46	0.83	0.18
5	0.62	0.64	0.64	0.64
6	1.60	1.38	0.42	0.46
7	0.94	0.36	0.32	0.28
8	1.08	1.74	0.46	0.36
9	0.64	0.32	0.68	0.02
10	0.98	0.56	0.42	0.46
11	0.48	0.46	0.32	0.26
12	1.67	0.90	0.46	0.36
13	1.28	0.86	0.38	0.34
14	1.52	1.02	0.22	0.28
15	0.78	0.72	0.48	0.28
16	0.80	0.64	0,38	0.26
17	0.86	0.86	0.30	0.16
18	1.80	0.96	0.38	0.18
19	1.56	0.96	0.40	0.26
20	0.84	0.38	0.36	0.28
Total	19.33	13.82	8.41	6.02
Average	0.97	0.69	0.42	0.30

Calculations for control limits.

$n = 4,\ A_2 = 0.73,\ D_3 = 0,\ D_4 = 2.28$

	UCL	LCL
	Before Control	
For $\bar{X}$	$\bar{\bar{X}} + A_2\ \bar{R}$	$\bar{\bar{X}} - A_2\ \bar{R}$
	$0.97 + 0.73 \text{ x } 0.69$	$0.97 - 0.73 \text{ x } 0.69$
	$= 1.47$	$= 0.47$
For R	$\bar{R} \text{ x } D_4$	$\bar{R} \text{ x } D_3$
	$0.69 \text{ x } 2.28 = 1.57$	$0.69 \text{ x } 0 = 0$
	After Control	
For $\bar{X}$	$0.42 + 0.73 \text{ x } 0.30$	$0.42 - 0.73 \text{ x } 0.30$
	$= 0.64$	$= 0.20$
For R	$0.30 \text{ x } 2.28 = 0.68$	$0.30 \text{ x } 0 = 0$

Both $\bar{X}$– chart and R – chart are very useful instruments for routine control and detection of troubles. But their use is limited to variables only. They need special measurements for controls. Indiscriminate use of these charts is often impracticable as well as uneconomical.


X and R-Control Charts for
Residual Chlorine in Pulp
X- Cl 2 Lbs/ TONNE
UCL=1.47
X̿=0.97
LCL=0.47
X̄-CHART
UCL= 0.64
X̿= 0.42
LCL= 0.20
BEFORE CONTROL
AFTER CONTROL
R- Cl 2 Lbs/ TONNE
UCL=1.67
R-CHART
R̄ = 0.69
UCL= 0.68
R̄ = 0.20
LCL= 0
LCL= 0

Attribute Control Charts :

It is applied to quality characteristic that can be observed as an attribute only. Often articles are classified as defective or non-defective even when they are measurable. Such inspection is referred as attribute inspection. The probabilities of various number of defectives in a sample 'n' follows a Binomial distribution. Like control charts for variables, here also control limits lie on either side of central line at a distance of three times the Standard Deviation. A 'Fraction Defective (np) chart' is based on the properties of Binomial distribution. Another important attribute chart is 'C—chart'. In this case C' is number of defects per unit articles, rather than proportion of defectives (p). It is based on Poisson Distribution. The control limits based on statistical calculations for two types of attribute control charts are as below :

Chart	u. c. L.	L. C. L.	Central line
p — chart	$p \div 3\sqrt{\frac{p\,(1-p)}{n}}$	$p - 3\sqrt{\frac{p\,(1-p)}{n}}$	p
c — chart	$\bar{C} \div 3\sqrt{\bar{C}}$	$\bar{C} - 3\sqrt{\bar{C}}$	$\bar{c}$

where p = fraction or proportion of defectives in a sample

$\bar{c}$ = Average number of defects in each unit of an article.

The $\bar{x}$, R, p & c — charts are the most convenient charts used in the industry. They can help in arriving at the causes of variation in quality of any product and hence guide to build up quality standards. They are most useful when implemented with the help of technicians who would be helpful in rectifying the possible defects in the process.