Mathematical Model of Recycling

HARI R. GOYAL

SUMMARY

The recycling system stabilisesitself after a few cycles and multiple Recycling of fibers do not affect the overall resulting furnish and strength properties.

Paper can be the best and most economical raw material for paper. Though this is not always true depending upon mill to mill and country to country. But there are no two thinking that paper as a raw material for paper reduces the load on the virgin fibre sources which are scarce in major part of the world. The central idea of this article is not to discuss the merits/demerits of recycling of fibres but to see the effect of multiple recycling of fibres on resulting paper that too theoreticaly-

It has been a question for paper makers that how many times fibers can be recycled and up to what fraction of furnish. The recycled fibers are in general weaker and slower in drainage than the virgin fibers and strength and freeness decreases with each cycle. This decrease in strength and freeness can be attributed to the cutting of fibers during winding, cutting, slitting, printing, converting, packaging, handling, using; collecting, repulping etc. etc. as well as aging of paper.

How many times fibers can be recycled depends largely on how much fibers we are recyling each times. It is practically impossible to collect all the paper back from the users. At one or the other stage it becomes uneconomical to collect the paper, so the first self imposed limit of amount of recycling is set by the constraint in collection. The second limit is set by the increase in demand of paper and this can be met only by adding virgin fibers. The third costraint may be that still paper is most convenient medium of storage of information. The stored paper will not be availabe for recycling. Moreover papers which are treated or coated with chemicals are generaly not suitable for recycling.

So these self imposed constraint of amount of recycled fibers available for use itself helps in recycling the fibers any number of times along with virgin fibers. Let us see how is this possible theoretically. Four formulii are mentioned below and the derivation is given in the APPENDIX–A

$$RCn = A^{**}n\ (1-A) \quad \text{Where } n = 1,2,3\ldots\ldots N-1$$
$$RCn = A^{**}n$$

$$\text{FURNISH} = A^{**}n + \sum_{i=O}^{n-1} A^{**}i\ (1-A) \quad (n = N)$$

$$\text{STRENGTH} = SFn.A^{**}n + \sum_{i=O}^{n-1} SFi.\ (1-A).\ A^{**}i \quad (n=N)$$

$$\text{RELATIVE STRENGTH} = 1 - \sum_{i=O}^{n} A^{**}i.(Ri-1\ -Ri)$$

Where A is the fraction of recycled fibers in furnish from last run.

n is number of times a fiber is recycled.

N is number of run.

RCn is fraction of fibers recycled n time.

SFn is strength of n times recycled fibeıs.

Rn is strength of n times recycled fiber relative to virgin fiber.

While deriving the formulii it is assumed that the recycling fraction is from the last run of the paper only. Though this is not strictly true but fairly justified and secondly this gives fibers recycled more numbers of times than any other combinatiou of furnish and hence repersents worst possible situation.

Graduate Students, 100 Wedein Avenue. E-.0,
KALAMAZOO MI 49008 - U S A

Now let us consider how and when a system will be stabilised. We are considering 3 fractions of recycled fibers as 25%, 40%, 50%,. Let N is 6. Practically N will be very high and will depend on the time lag between PRODUCTION - CONSUMPTION RECOLLECTION. A system will be considered as stabilised when the fraction of higher number recycling is less than 1%.

FURNISH

	A =	25%	40%	50%
RCO	= 1—A	0.7500	0.6000	0.5000
RCI	= A.(1—A)	0.1875	0 2400	0.2500
RC2	= A**2.(1—A)	0.0468	0.0960	0.1250
RC3	= A**3.(1—A)	0.0117	0.0384	0.0625
RC4	= A**4.(1—A)	0.0029	0.0154	0.03125
RC5	= A**5.(1—A)		0.0061	0.01562
RC6	= A**6.(1—A)			0.0078

Increase in N increases the number of times the recycled fibers in the furnish but their fraction decreases very rapidly. The system stabilise after 4th run for 25% recycling fraction, after 5th run for 40% and after 6th run for 50%. and so on.

STRENGTH :

Let us consider a reduction in strength by 10% and 15% after each cycle. Though both figures of 50% recycling and 15% strength reduction are higher but on safer side.

RECYCLING FRACTION	STRENGTH	10% 15%	RECYCLING FRACTION 25%	40%	50%
RC0 = (1—A)	SFO	1.0/1.0	0.750/0.750	0.600/0.600	0.500/0.500
RC1 = A (1—A)	SF1	0.9/.85	0.169/0.159	0.216/0.204	0.225/0.212
RC2 = A**2 (1—A)	SF2	0.8/0.7	0.037/0.033	0.077/0.067	0.100/0 087
RC3 = A**3 (1—A)	SF3	0,7/.55	0.008/0.006	0.027/0.021	0.044/0 034
RC4 = A**4 (1—A)	SF4	0.6/0.4		0.009/0.006	0.019/0.013
RC5 = A**5 (1—A)	SF5	0.5/.25			0.008/0.004
		TOTAL	0.964/0.948	0.926/0.898	0.896/0.850

Thus we see that for 10% strength reduction per cycle the overall strength is more than 96%, 93% and 90% for 25%, 40% and 50% recycling respectively. Similarly for 15% strength reduction consideration for each cycle, the overall strength is above 95%, 90% and 85% for above three recycling fraction.

CONCLUSION :

In the Situation of Self Impossed Limitation of Recycling Fiber Fraction there is no danger of so Called' Fiber Contamination' or Drastic Strength Reduction With Increasing Usage of Recycled Fibers.