Simulation of Multi-Effect Evaporator for Paper Mill-Effect of Flash and Product Utilization for Mixed Feeds Sequences

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Evaporators are widely used in the chemical process industries (paper, sugar, glycerin etc.)to concentrate a solution, to recover solvents (optional) or to reuse the heat of the solvent in the same or ancillary units. A generalized algorithm was earlier developed (1)for design and simulation of long tube black liquor multiple effect evaporator system (MEE) with mixed feed sequence in pulp & paper industry. In this present investigation the above algorithm is extended for mixed feed with condensate and product flash for additional advantages of energy gains and higher steam economy in mixed feed sequence. Models are based on steady state mass balance, enthalpy balance and beat transfer rate equations interlinked with over all heat transfer coefficient obtained from Gudmundson's model (2) and suitable correlations of physico- thermal properties of black liquor as well as the BPR values as a unction of temperature and concentration which are proposed by Ray et. al. (3). A program is developed in FORTRAN- 77 language. The numerical technique used is Newton- Raphson method with Jacobian matrix & methods of Gauss Elimination with partial pivoting supplemented with LU decomposition with the aid of Hilbert Norms.

INTRODUCTION

Mathematical modelling is an indispensable tool, now a days, to analyze, simulate, optimized and finally control of any chemical process system. Pulp and Paper industry is not an exception. To improve the energy efficiency single effect evaporators are connected in series, each operating at successively lower pressure. Ray(4), Mathur (5) & Singh (6) have shown in detail for black liquor evaporation, the variation of total annual cost model as a function of effects. After optimization of the single valued of objective function the number of effects have been found to be six for black liquor concentration in Indian Mills at the prevaling cost.

Multiple effect evaporator may be arranged in many ways, namely, forward, backward, mixed feed & split feed sequences. In this present investigation, mixed feed sequence (5→6→4→3→2→1) has been taken where the feed enters in 5th effect as shown in diagram-1 (Appendix-A). This mixed feed design is usually used in Indian mill with the option of feeding to any body in between first and last. Feeding to fifith body is one option. Process steam is provided to the first effect. Water vapor from this effect is used in turn to heat the second and so on to the last. All the evaporator stages are having same heat transfer area, of identical design and terminal conditions and operate with fixed flow rates.

There are a very few models available on evaporation operation for black liquor concentration (1 - 15). Kern (7) has given a model for concentration of commercial soda the black liquor from a paper mill. Later Mathur (5), Goel (8) also developed models & solved the problems for kraft black liquor concentration using the procedures documented in Holland (9). All the above works suffer from many limitations, someway or the other and do not, in fact, take care of simulation of multiple effect evaporator system in pulp & paper industry. In Mathur's works condensate flash utilization is not considered whereas Goel's design is based on backward feed sequence with no return of condensate from first body which is in fact invariably used as boiler feed water. Though Kern's model is good, a very weak soda liquor is used in the system instead of kraft black liquor as feed to a sextuple effect evaporator system. In addition, the problem has been solved by trial and error (Badger-McCabe) method. There are limitations of this method as every time when there are changes of sequences or variation in input data, exhaustive trial and error calculation need to be done. No general correlation of physico-thermal properties and overall heat transfer coefficient (U) are used. Hence it can not be employed for general design. Models for various feed sequences are developed by Ray(4), Mathur(5), Singh(6) & Sharma(10). Thses are validated with the Kern's data for soda liquor for a system of six body evaporator. The systems are simulated with the range of input data globally used by the pulp & paper industry including India. Some of the results are already published in literature (1,12,13,14). Bremford et. al. (16) has developed an improved steady state models for multiple effect evaporator of Carter Holt Harvery Kinleith mill having integral after/pre heaters. The models were compared with plant data from the mill. Thses however give predicts with average errors in mass & energy balance between 14% to 23%. The model building is based on a fixed configuration of a mill suiting to their requirements. This is solved by iterative/modular strategies. In this present investigation, a conventional type of mixed feed sequence with various options without any integral heaters is presented. The options consist of the mixed feed with and without condensate & product flash. A comparison among the various options for steam consumption, steam economy and heating surface area is to be examined. Therefore, the present study is an extension of the pervious works. Correlations of physico-thermal properties are taken from Ray et. al. (3) and over all heat transfer coefficient from Gudmundson's model (2) for simulation of long tube vertical multiple effect evaporator system for black liquour concentration.

Modeling

Modelling of mixed feed sequence is not available in literature is details though all most all MEE in India works on these sequences. Therefore, it is an imperative meaning to develop models on these type of sequences. As a preliminary investigation a mixed feed sequence ($5\rightarrow6\rightarrow4\rightarrow3\rightarrow2\rightarrow1$) is attempted withot imtroducing any internal and external heaters.

The following sections deal with a step by step procedure for model building of a sextuple effect long tube vertical evaporator system. The schematics of the MEE is shown in diagram 1.

Mathematical Model for Multiple Effect Evaporator System for Mixed Feed Sequence with no Flash

($5\rightarrow6\rightarrow4\rightarrow3\rightarrow2\rightarrow1$)

The prsent mathematical model is established based on scaling procedure of Holland (9) for miltiple effect evaporator system in pulp and paper industry. Five independent equations can be written for each effect; based on mass balance, a component material balance, an enthalpy balance, a heat transfer rate equation and the over all mass balance as follows. The models give rise to a set of 12 unknown independent non - linear algebraic equations from in 12 unknowns if there is no flashing either from condensate or from product.

For Ist effect of Evaporator

a. Mass Balance

$$L_2 = V_1 + L_1 \qquad ...(1)$$

b. Component Balance

$$L_2\ X_2 = L_1X_1 \qquad ...(2)$$

c. An Enthalpy Balance

$$Q_1 = V_0\ H_0 - C_1hc_1$$
$$= V_0\ (hc_1+\lambda_0+CP_VBPR_0) - C_1hc_1$$
$$= V_0\ (\lambda_0+CP_VBPR_0) \qquad ...(3)$$

d. Heat Transfer Rate

$$Q_1 = U_1\ A_1\ (\Delta T_1)_{eff}$$
$$= U_1\ A_1\ (T_0-T_1BPR_1)$$
$$\text{Where, } (\Delta T_1)_{eff} = \Delta T_1-BPR_1 \qquad ...(4)$$

e. Total mass balance on steam chest

$$C_1 = V_0 \qquad ...(5)$$

An enthalpy balance on the process

$$L_2\ h_2 + Q_1 = V_1H_1 + L_1h_1 \qquad ...(6)$$

From the equation (1) and (3), Eq. (6) gives

$$L_2\ (h_2 + h_1) + V_0H_0 - C_1hc_1\ (L_2 - L_1)\ (H_1-h_1) = 0 \qquad ...(7)$$

Or $L_2\{CP_2(T_2+BPR_2)-CP_1(T_1+BPR_1)\}+V_0(\lambda_0+CP_V$ $BPR_0)-(L_2-L_1)\{(\lambda_1+CP_VBPR_1)-CP_1(T_1+BPR_1)\}$ $-(L_2-L1)(\alpha T1+\beta)=0$ ...(8)

Where $h_i=CP_i(T_i+BPR_i)$, $H_i=hc_{i+1}+\lambda i+CP_VBPR_i$, $hc_{i+1}=(\alpha T_i+\beta)$

And the value of constants a and b are 4.1832 and 0.127011 respectively.

Now f_1 is defined as;

$$f_1=L_2\{CP_2(T_2+BPR_2)-CP_1(T_1+BPR_1)\}+V_0(\lambda_0+CP_V BPR_0)-(L_2-L_1)\{(\lambda_1+CP_VBPR_1)-CP_1(T_1+BPR_1)\}-(L_2-L_1)(\alpha T_1+\beta) \quad ...(9)$$

From the eqs.(3) and (4) gives

$$U_1A_1(T_0-T_1-BPR_1)-V_0(\lambda_0+CP_VBPR_0)=0 \quad ...(10)$$

f_2 is defined as

$$f_2=U_1A_1(T_0-T_1-BPR_1)-V_0(\lambda_0+CP_VBPR_0) \quad ...(11)$$

After scaling procedure based on Holland (9) the functions will be in the form as follows;

$$g_1=l_2\ T_0\{CP_2(u_2+BPR_2/T_0)-CP_1(u_1+BPR_1/T_0)\}/\lambda_0+v_0(\lambda_0+CP_VBPR_0)/\lambda_0-(l_2-l_1)\{(\lambda_1+CP_VBPR_1)-CP_1T_0(u_1+BPR_1/T_0)\}/\lambda_0-(l_2-l_1)(\alpha u_1T_0+\beta)/\lambda_0 \quad ...(12)$$

$$g_2=U_1a_1T_0(l-u_1-BPR_1/T_0)/50\ \lambda_0-v_0(\lambda_0+CP_V BPR_0)/\lambda_0 \quad ...(13)$$

For IInd and IIIrd Effect

$$g_i=l_{n+1}\ T_0\{CP_{n+1}(u_{n+1}+BPR_{n+1}/T_0)-CP_n(u_n+BPR_n/T_0)\}/\lambda_0+(l_n-l_{n-1})(\lambda_{n-1}+CP_VBPR_{n-1})/\lambda_0-(l_{n+1}-l_n)\{(\lambda_n+CP_VBPR_n)-CP_nT_0(u_1+BPR_1/T_0)\}/\lambda_0-(l_2-l_1)(\alpha u_1T_0+\beta)/\lambda_0 \quad ...(14)$$

$$g_{i+1}=U_na_nT_0(u_{n-1}-u_nBPR_n/T_0)/50\ \lambda_0-(l_n-l_{n-1})(\lambda_{n-1}+CP_V BPR_{n-1})/\lambda_0 \quad ...(15)$$

for i= 3, 5; n= 2, 3

For IVth Effect

$$g_7=l_6\ T_0\{CP_6(u_6+BPR_6/T_0)-CP_4(u_4+BPR_4/T_0)\}/\lambda_0+(l_4-l_3)(\lambda_3+CP_VBPR_3)/\lambda_0-(l_6-l_4)\{(\lambda_4+CP_VBPR_4)-CP_4T_0(u_4+BPR_4/T_0)\}/\lambda_0-(l_6-l_4)(\alpha u_4T_0+\beta)/\lambda_0 \quad ...(16)$$

$$g_8=U_4a_4T_0(u_3-u_4-BPR_4/T_0)/50\ \lambda_0-(l_4-l_3)(\lambda_3+CP_V BPR_3)/\lambda_0 \quad ...(17)$$

For Vth Effect

$$g_9=T_0\{CP_f\,u_f-CP_5(u_5+BPR_5/T_0)\}/\lambda_0+(l_6-l_4)(\lambda_4+CP_V BPR_4)/\lambda_0-(l-l_5)\{(\lambda_5+CP_VBPR_5)-CP_5T_0(u_5+BPR_5/T_0)\}/\lambda_0-(l-l_5)(\alpha u_5T_0+\beta)/\lambda_0 \quad ...(18)$$

$$g_{10}=U_5a_5T_0(u_4-u_5-BPR_5/T_0)/50\ \lambda_0-(l_6-l_4)(\lambda_4+CP_V BPR_4)/\lambda_0 \quad ...(19)$$

For VIth Effect

$$g_{11}=l_5\ T_0\{CP_5(u_5+BPR_5/T_0)-CP_6(u_6+BPR_6/T_0)\}/\lambda_0+(l-l_5)(\lambda_5+CP_VBPR_5)/\lambda_0-(l_5-l_6)\{(\lambda_6+CP_VBPR_6)-CP_6T_0(u_6+BPR_6/T_0)\}/\lambda_0-(l_5-l_6)(\alpha u_6T_0+\beta)/\lambda_0 \quad ...(20)$$

$$g_{12}=U_6a_6T_0(u_5-u_6-BPR_6/T_0)/50\ \lambda_0-(l-l_5)(\lambda_5+CP_V BPR_5)/\lambda_0 \quad ...(21)$$

Mathematical Model for Multiple Effect Evaporator System with Mixed Feed Sequence with Condensate Flash Tank

(5→6→4→3→2→1)

For Ist Effect

$$g_1=l_2\ T_0\{CP_2(u_2+BPR_2/T_0)-CP_1(u_1+BPR_1/T_0)\}/\lambda_0+v_0(\lambda_0+CP_VBPR_0)/\lambda_0-(l_2-l_1)\{(\lambda_1+CP_VBPR_1)-CP_1T_0(u_1+BPR_1/T_0)\}/\lambda_0-(l_2-l_1)(\alpha u_1T_0+\beta)/\lambda_0 \quad ...(22)$$

$$g_2=U_1a_1T_0(l-u_1-BPR_1/T_0)/50\ \lambda_0-v_0(\lambda_0+CP_V BPR_0)/\lambda_0 \quad ...(23)$$

For IInd and IIIrd Effect

$$g_i=l_{n+1}\ T_0\{CP_{n+1}(u_{n+1}+BPR_{n+1}/T_0)-CP_n(u_n+BPR_n/T_0)\}/\lambda_0+(l_n-l_{n-1}+m_{n-1})(\lambda_{n-1}/\lambda_0)+(l_n-l_{n-1})CP_VBPR_{n-1}/\lambda_0-(l_{n+1}-l_n)\{(\lambda_n+CP_VBPR_n)-CP_nT_0(u_n+BPR_n/T_0)\}/\lambda_0-(l_{n+1}-l_n)(\alpha u_nT_0+\beta)/\lambda_0 \quad ...(24)$$

$$g_{i+1}=U_na_nT_0(u_{n-1}-u_n-BPR_n/T_0)/50\ \lambda_0-(l_n-l_{n-1}m_{n-1})(\lambda_{n-1}/\lambda_0)-(l_n-l_{n-1})CP_VBPR_{n-1})/\lambda_0 \quad ...(25)$$

Where i = 3, 5; n = 2, 3

For IVth Effect

$$g_7=l_6\ T_0\{CP_6(u_6+BPR_6/T_0)-CP_4(u_4+BPR_4/T_0)\}/\lambda_0+(l_4-l_3+m_3)(\lambda_3/\lambda_0)+(l_4-l_3)CP_VBPR_3)/\lambda_0-(l_6-l_4)\{(\lambda_4+CP_VBPR_4)-CP_4T_0(u_4+BPR_4/T_0)\}/\lambda_0-(l_6-l_4)(\alpha u_4T_0+\beta)/\lambda_0 \quad ...(26)$$

$$g_8=U_4a_4T_0(u_3-u_4-BPR_4/T_0)/50\ \lambda_0-(l_4-l_3)(\lambda_3+CP_V BPR_3)/\lambda_0 \quad ...(27)$$

For Vth Effect ;

$g_9=T_0\{CP_f\,u_f-CP_5(u_5+BPR_5/T_0)\}/\lambda_0+(l_6-l_4+m_4)(\lambda_4/\lambda_0$

$) + (l_6 - l_4)\, CP_v\, BPR_4/\lambda_0 - (1 - l_5)\, \{(\lambda_5 + CP_v\, BPR_5) - CP_5 T_0(u_5 + BPR_5/T_0)\} / \lambda_0 - (1 - l_5)\, (\alpha u_5 T_0 + \beta)/\lambda_0$ ...(28)

$g_{10} = u_5\, a_5 T_0(u_4 - u_5 - BPR_5/T_0) / 50\lambda_0 - (l_6 - l_4 + m_4)\, (\lambda_4/\lambda_0) - (l_6 - l_4)\, CP_v\, BPR_4/\lambda_0$ ...(29)

For VIth Effect ;

$g_{11} = l_5 T_0\{CP_5\, (u_5 + BPR_5/T_0) - CP_6(u_6 + BPR_6/T_0)\} / \lambda_0 + (1 - l_5 + m_5)(\lambda_5/\lambda_0) + (1 - l_5)\, CP_v\, BPR_5/\lambda_0 - (l_5 - l_6)\, \{(\lambda_6 + CP_v\, BPR_6) - CP_6 T_0(u_6 + BPR_6/T_0)\} / \lambda_0 - (l_5 - l_6)\, (\alpha u_6 T_0 + \beta)/\lambda_0$ ...(30)

$g_{12} = u_6\, a_6 T_0(u_5 - u_6 - BPR_6/T_0) / 50\lambda_0 - (1 - l_5 + m_5)\, (\lambda_5/\lambda_0) - (1 - l_5)\, CP_v\, BPR_5/\lambda_0$ ...(31)

For Ist Condensate Flash Tank :

(a). Mass and energy balance equations for Ist condensate flash tank :"

Total mass balance equation

$C_1 = M_1 + C_{11}$

Enthalpy balance

$C_1\, hc_1 = M_1\, Hs_1 + C_{11}\, hc_{11}$

$C_1\, hc_1 = M_1\, Hs_1 + C_{11}\, hc_2$ $[\, hc_{11} = hc_2]$

$C_1\, hc_1 = M_1\, Hs_1 + (C_1 - M_1)\, hc_2$

$C_1\, (hc_1 - hc_2) = M_1\, (Hs_1 - hc_2)$ ...(32)

Based on the linear regression equations from steam table the enthalpy functions for both liquid condensate and steam are given as under :

$hcj = \alpha T_{j-1} + \beta$ $Hsj = \gamma T_j + \delta$

where the values of α, β, γ and δ are 4.1832, 0.127011, 1.75228 and 2503.35 respectively, and

$C_1 = V_0$ hence eq. 24 becomes

$V_0\, \alpha\, (T_0 - T_1) - M_1(k_1 T_1 + k_2) = 0$

Where $k_1 = \gamma - \alpha$ and $k_2 = \alpha - \beta$

In functional notations we can write

$f_{13} = V_0\, \alpha(T_0 - T_1) - M_1\, (k_1\, T_1 + k_2)$ ...(33)

After scaling procedure based on Holland (9) the function will be in the form as follows ;

$g_{13} = V_0\, \alpha\, T_0\, (1 - u_1)/\lambda_0 - m_1\, T_0\, (k_1\, u_1 + k_2/\, T_0) / \lambda_0$ ...(34)

For IInd to IVth Condensate Flash Tank :

$g_i = (l_n - l_1 + m_l)\, \alpha\, T_0\, (u_{n-1} - u_n)/\lambda_0 - m_n\, T_0\, (k_1\, u_n + k_2/\, T_0) / \lambda_0$ ...(35)

(i =14,15,16; n=2,3,4)

For Vth Condensate Flash Tank :

$g_{17} = (l_6 - l_1 + m_l)\, \alpha\, T_0\, (u_4 - u_5)/\lambda_0 - m_5\, T_0\, (k_1\, u_5 + k_2/\, T_0)\, \lambda_0$ ...(36)

Mathematical Model for Multiple Effect Evaporator System with Mixed Feed Squence with Condensate Flash and Product Flash Tank(5→6→4→3→2→1) :

For the design procedure of a multiple effect evaporator system with mixed feed with flashing of condensate as well as with the product flash, the model give rise to a set of 17 non-linear algebraic equations with 17 unknowns, where is the product flash amount is added in the third effect. Note that there are no changes in the model equations from Ist to IIIrd effect, Vth and VIth effect. However, there will be changes of models in the IVth body. The equations for condensate flash will also be changed like the equation for IVth effect. These are given as follows.

For IVth Effect ;

$g_7 = l_6 T_0\{CP_6\, (u_6 + BPR_6/T_0) - CP_4(u_4 + BPR_4/T_0)\} / \lambda_0 + (l_4 - l_3 + m_3)(\lambda_3/\lambda_0) + (l_4 - l_3)\, CP_v\, BPR_3/\lambda_0 - (l_6 - l_4)\, \{(\lambda_4 + CP_v\, BPR_4) - CP_4 T_0(u_4 + BPR_4/T_0)\} / \lambda_0 - (l_6 - l_4)\, (\alpha u_4 T_0 + \beta)/\lambda_0$ ...(37)

$g_8 = u_4\, a_4 T_0(u_3 - u_4 - BPR_4/T_0) / 50\lambda_0 - (l_4 - l_3 + m_5)\, (\lambda_3/\lambda_0) - (l_4 - l_3)\, CP_v\, BPR_3/\lambda_0$ ...(38)

For IVth Condensate Flash Tank :

$g_{16} = (l_4 - l_1 + m_1 + m_p)\, \alpha\, T_0\, (u_{n-1} - u_n)/\lambda_0 - m_4\, T_0\, (k_1\, u_n + k_2/\, T_0)/\lambda_0$ ...(39)

Search for the Solution of Models:

Numerical techniques, available for solving linear equations are Cramer's rule, Gauss Elimination, Square Root Method, Scheme of Cholesky and Iterative methods, like Gauss Seidel method and Gauss Jordan Method etc. The method for solving non-linear equations are limited. Most popular are Newton's Method and method of iteration. Newton's algorithm is widely used because, at least, in near neighbourhood of a root, it more rapidly converges than other methods. However, to establish the rapidity of convergence of the Newton' s process and the uniqueness of the root of the system and the stability of the process with respect to choice of the initial approximation, certain modifications are made; these are Newton-Raphson, Kantovich, Jacobi method, Broyden method, Secent Method, Steepest Decent method (Gradient method) and Quasi Newton method. Holland (9) have reported that among the above methods, the Newton- Raphson-Jacobi matrix method followed by method of Gauss elimination is the best to solve the system of non linear equations. The methodology has been examplifled with caustic soda solutions limiting to "triple effect only. Based on the suggested procedure, modeling for five effect evaporator system for sugar juice has been designed (15). The details of how Newton- Raphson method can be employed for black liquor evaporator design with mixed feed sequence is given below.

Newton-Raphson Method:

The extension of the method of Newton to functions of several variables is called the Newton-Raphson method (9). It is developed in manner analogous to that known for Newton's method. The Newton-Raphson method consists of the repeated use of the linear terms of the Taylor series expansion of the function $g_i, g_2, g_3 \ldots\ldots g_{12}$.

$$0 = g_i + (\partial g_i/\partial V_0)\Delta v_0 + (\partial g_i/\partial u_1)\Delta u_1 + (\partial g_i/\partial u_2)\Delta u_2 + (\partial g_i/\partial u_3)\Delta u_3 + (\partial g_i/\partial u_4)\Delta u_4 + (\partial g_i/\partial u_5)\Delta u_5 + (\partial g_i/\partial l_2)\Delta l_2 + (\partial g_i/\partial l_3)\Delta l_3 + (\partial g_i/\partial l_4)\Delta l_4 + (\partial g_i/\partial l_5)\Delta l_5 + (\partial g_i/\partial l_6)\Delta l_6 + (\partial g_i/\partial a)\Delta a \quad \ldots(40)$$

$$(i = 1,2,3,\ldots\ldots\ldots 11,12)$$

where

$\Delta v_0 = v_{0(k+1)} - v_{0k}$ $\Delta l_2 = l_{2(k+1)} - l_{2k}$

$\Delta u_1 = u_{1(k+1)} - u_{1k}$ $\Delta l_3 = l_{3(k+1)} - l_{3k}$

$\Delta u_2 = u_{2(k+1)} - u_{2k}$ $\Delta l_4 = l_{4(k+1)} - l_{4k}$

$\Delta u_3 = u_{3(k+1)} - u_{3k}$ $\Delta l_5 = l_{5(k+1)} - l_{5k}$

$\Delta u_4 = u_{4(k+1)} - u_{4k}$ $\Delta l_6 = l_{6(k+1)} - l_{6k}$

$\Delta u_5 = u_{5(k+1)} - u_{5k}$ $\Delta a = a_{(k+1)} - a_k$

and where the subscripts are k^{th} and $(k+l)^{st}$ trials. The twelve non-linear equations may be stated in compact form by means of the given matrix equation.

$$J_k \Delta X_k = -g_k \quad \ldots(41)$$

Where J_k is called the Jacobian matrix of order 12. ΔX_k and g_k are conformable vectors. A display of the elements of J_k, g_k and X_k follows ;

The subscripts k and k+ 1 denote that elements those for k^{th} and $k+1^{st}$ trials respectively. On the basis of assumed (initial) set of values for the elements of the column matrix X_k (= X_0) the corresponding values of the elements of J_k and g_k are computed.

$$X_k = [v_0 \;\; u_1 \;\; u_2 \;\; u_3 \;\; u_4 \;\; u_5 \;\; l_2 \;\; l_3 \;\; l_4 \;\; l_5 \;\; l_6 \;\; a]^T$$

The computed values of J_k and g_k are used to calculate the values of ΔX_k by solving eq. (41) using Gaussian Elimination method with partial pivoting supplemented with LU decomposition. Subsequently, X_{k+1} is calculated from eq.(42) for next trial. The procedure is repeated until the desired accuracy of the values of unknown variable are obtained. For the present work, the Jacobian matrix is generated numerically. For this, a perturbation of the magnitude 0.001 in column vector X_k is applied and subsequent values of g_k are computed to form J_k. The tolerance selected for variables $v_0, u_1, u_2, u_3, u_4, u_5, l_2, l_3, l_4, l_5, l_6$ and a, relating to the functions g_1 to g_{12} is taken to be 0.000001 for an acceptable solution.

Computer Simulation :

A computer program based on FORTRAN- 77 have been developed, using the mathematical techniques, indicated above.

An initial approximation; the evaporation rates and area are considered equal as :

$$v_j = v \qquad (l \geq j \leq 6)$$

$$J_k = \begin{pmatrix} \partial g_1/\partial v_0 & \partial g_1/\partial u_1 & \partial g_1/\partial u_2 & \ldots\ldots\partial g1/\partial l_6 & \partial g_1/\partial a \\ \partial g_2/\partial v_0 & \partial g_2/\partial u_1 & \partial g_2/\partial u_2 & \ldots\ldots\partial g_2/\partial l_6 & \partial g_2/\partial a \\ \partial g_3/\partial v_0 & \partial g_3/\partial u_1 & \partial g_3/\partial u_2 & \ldots\ldots\partial g_3/\partial l_6 & \partial g_3/\partial a \\ ------ & ------ & ------ & \ldots\ldots------ & ------ \\ ------ & ------ & ------ & \ldots\ldots------ & ------ \\ ------ & ------ & ------ & \ldots\ldots------ & ------ \\ \partial g_{12}/\partial v_0 & \partial g_{12}/\partial u_1 & \partial g_{12}/\partial u_2 & \ldots\ldots\partial g_{12}/\partial l_6 & \partial g_{12}/\partial a \end{pmatrix}$$

and $gk = [g_1 \;\; g_2 \;\; g_3 \;\; g_4 \ldots\ldots g_{12}]^T$

$$\Delta X = Xk+1-Xk = [\Delta v0 \;\; \Delta u_1 \;\; \Delta u_2 \;\; \Delta u_3 \;\; \Delta u_4 \;\; \Delta u_5 \;\; \Delta l_2 \;\; \Delta l_3 \;\; \Delta l_4 \;\; \Delta l_5 \;\; \Delta l_6 \;\; \Delta a]^T \ldots\ldots (42)$$

Table 1 : Physico-Thermal Properties

Parameters	Correlation	Function of
Latent heat of vaporization, λ, J/kg	λ = 2519.5- 2.653 x T	Temperature
Boiling point rise,BPR, K	BPR = 41.4 x(TS +1)^2	Concentration
Specific Heat, C_p, J/kg K	C_p = { 1.0 -3.234 x (TS/TI)} x 4190 Where T1 = (T x 1.8 +32)	Temperature and Concentration
Thermal conductivity, k, W/Mk	k=(0.504-0.282xTS+ 1.35x 1 0^{-3})x 1.163	Concentration
Pressure, P, kN/m^2	P= 3.73812-0.108896xT1 +0.00122806xT1^2-6.69111xT^3 + 1.99203x 10^{-08}xT1^4 Where T1 = (T x 1.8 +32)	Temperature
Density, p, kg/m^3	p= 1007 -0.495xT +6.0x 18	Temperature and Concentration
Viscosity, μ, mPas	μ=exp[a+b(TS)+c(TS)^2 +d(TS)^3 Where a= 0.4717-0.02472xT +0.7059x10^{-05}xT^2 b=0.06973-0.5452x 10^{-3} +0.1656x 10^{-5}xT^2 c=0.002046+0.3183x 10^{-4} + T +0.9761 x 10-7xT^2 d=0.5793x 1 0^{-4}-0.1629x 10^{-8}xT^2	Temperature and Concentration

$a_i = a \qquad (l \geq j \leq 6)$

The variables, normally known, are; liquor feed rate, F, liquor feed temperature, T_f, liquor feed concentration, X_f, steam temperature, T_s, last effect temperature, T_n, and final product concentration X_p (=X_1) whereas specified variables are overall heat transfer coefficient U_1, U_2, U_3, U_4, U_5, U_6 and boiling point rise (BPR), BPR_1, BPR_2, BPR_3, BPR_4, BPR_5, BPR_6. The variables, which are unknown, are scaled vapor flow rate v_0, scaled liquor flow rate l_2, l_3, l_4, l_5, and l_6, scaled temperature u_1, u_2, u_3, u_4, u_5, fractional heating area, a and concentration terms X_2, X_3, X_4, X_5 and X_6.

In the set of 12 non-linear equations, the specified variables are; latent heat of vaporization (λ), boiling point rise (BPR) and specific heat (C_p) etc. These variables can be calculated from different correlations given in Table-1.

Operating Conditions :

As indicated, present mathematical models consisting of 12 or 17 non-linear algebraic equations are difficult to solve to achieve reasonable accuracy within limited time. However, using the same technique, it is possible to solve even these set of non-linear equations. For simulation purpose of different cases the following data are used. These range of data are generally used globally in the pulp & paper mill evaporator simulation.

The above set of data therefore agrees quite well within the limit of present practice in Indian pulp & paper mill. The results of simulation are discussed here under the following options.

Table 2 : Parameters for MEE systems

Parameter	Range	Target Value
Steam Temperature, T_s, °C	130-140	137.78
Feed Temperature, T_f, °C	70-80	76.67
Last Effect Temperature, T_n, °C	50-60	51.67
Feed Concentration, X_f	0.12-0.16	0.152
Feed flow Rate, F, kg/s	18-28	18.144

Options :

.A' → Mixed feed only

.A'_{CF} → Mixed feed with condensate flash utilization

.A'_{CPF} → Mixed feed with condensate & product flash utilization

RESULTS AND DISCUSSION :

Comparison of Steam Consumption, SC, Steam Economy, SE and Area,A.

Fig. 1-15 are plotted for mixed feed sequence without any flash (A'), mixed feed with condensate flash tank (A'_{CF}) & mixed feed with condensate and product flash tank (A'_{CPF}) to show the effect of various input parameters, namely, steam temperature, T_s, feed temperature, T_f, last effect temperature, T_n, feed concentration, X_f, and feed flow rate, F on SC, SE and A.

Effect of Steam Temperature

The effect of steam temperature on SE, SC & A are shown in figs.1-3. Following observations are made from the figures.

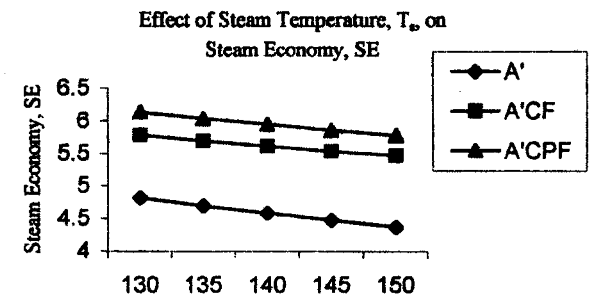

Fig. 2

From fig-1, SC increases in linear manner with the increases of steam temperature but with different slopes for all three options as indicated above in sec 3.3. SE also decreases nearly in the same manner. The area, A sharply decreases with the increase of steam temperature. It indicates that increase of steam temperature will benefit in terms of capital investment but from energy point of view it is undesirable. It is calculated that 10°C (130°C→140°C) rise in steam temperature gives reduction in steam economy, steam consumption and area by 4.715% , 4.995% and 21.623% (approx.) respectively. The steam consumption, SC drops to a value, on an average of 17.5% from A'→A'_{CF}, 5.56% from A'_{CF}→A'_{CPF} and 22.14% from A'→A'_{CPF}. Enhancement of steam economy are found to be of order of 21.24%, if one changes from A'→A'_{CF}, 5.91% from A'_{CF}→A'_{CPF} and 28.39% from A'→A'_{CPF}. The gain in economy and steam saving are quite large in proportion when compared to the extra heating surface area, which is of order of 9.01 %, for the changes from A'→A'_{CF},1.64%fromA'_{CF}→A'_{CPF} and 10.8 % from A'→A'_{CPF} indicating the scoring over the loss due to capital cost by the gain due to energy savings.

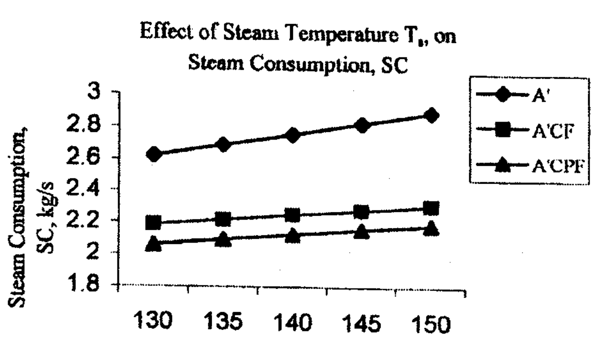

Fig. 1

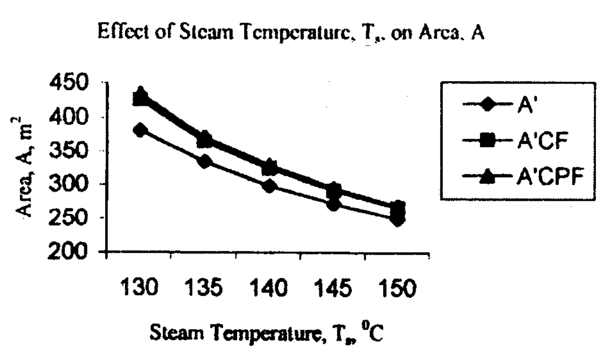

Fig. 3

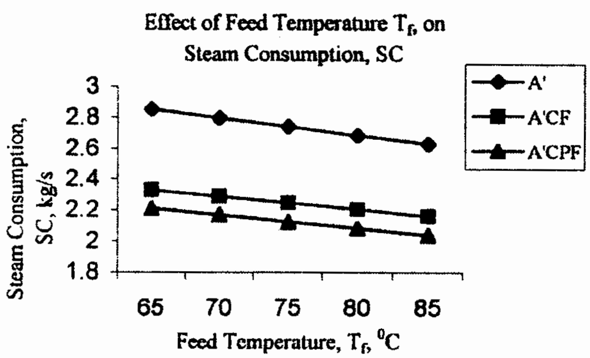

Fig. 4

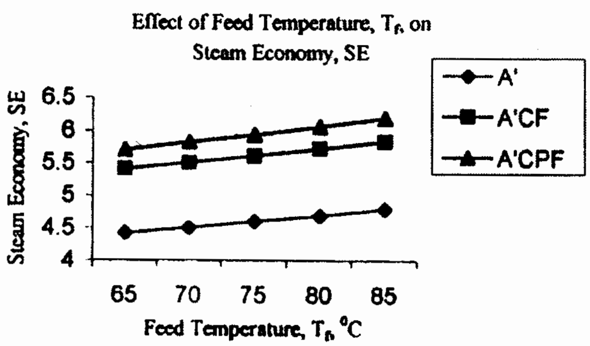

Fig. 5

Influence of Feed Temperature

Influences of feed temperature are given by the Fig. 4-6. From the figures, it is clear that Steam economy, SE increases linearly with the increase of feed temperature, decrease both SC and A. Therefore change of feed temperature is extremely desirable. All benefits can be accrued out of increase of feed temperature. In industry , it should be the attempt to either to preheat the liquor by externally heated steam in an external heater or in an integral heater of the evaporator or to restore temperature of black liquor from pulp mill. No loss of heat in the storage tank and from transportation lines to evaporator is allowed. Utmost care should be taken to avoid heat loss. With this exercise temperature above 90°C is possible to maintain easily. Many industries heat the liquor to compensate the loss of heat to get the benefit of higher temperature.

With the increase of T_f by 10°C (75°C→85°C), the SE increases by 4.14% and SC and area, A decrease by 3.9% and 2.04% (approx.) respectively.

If one proceeds from $A' \rightarrow A'_{CF}$, there is dramatic enhancement of SE noticed (21.9%), from $A'_{CF} \rightarrow A'_{CPF}$

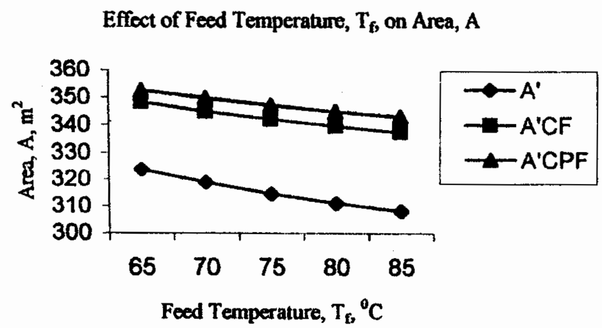

Fig. 6

5.76% only & from $A' \rightarrow A'_{CPF}$, 28.99% for T_f at 75°C. This amply clears that ehormous gain in economy from flashing of condensate and product can be obtained. The saving of SC if one desires to go from $A' \rightarrow A'_{CF}$, an extent of 18.04% is anticipated. It is interesting to note that if feed temperature is increased by 10°C (at T_f =75°C) the quantum of saving of SC from sequence $A' \rightarrow A'_{CPF}$ remains around 22.51% indicating that there is much gain obtained percentage wise and also a considerable saving when estimating steam consumption on an annual basis for an industry .This, in turn, earns a lot of profit.

There is approximate requirement of extra 8.71% of A for changes in configuration from $A' \rightarrow A'_{CF}$,1.47% from $A'_{CF} \rightarrow A'_{CPF}$, & 10.3% $A' \rightarrow A'_{CPF}$, for T_f at 75°C.

Variation of Last Effect Temperature

Variation of steam consumption, SC, steam economy, SE and A, area demand with the variation of last effect temperature, T_n, and the alternative designs (A', A'_{CF} & A'_{CPF}) are depicted in fig. 7-9. It is evident from the plots that there are displays of opposite trend of linearity by SC and SE values but upward concavity for variation

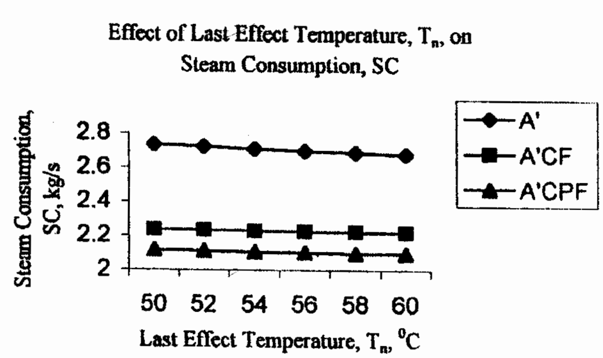

Fig. 7

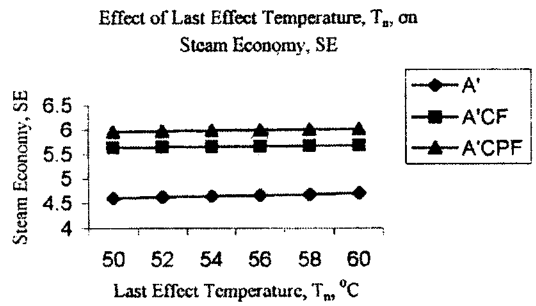

Fig. 8

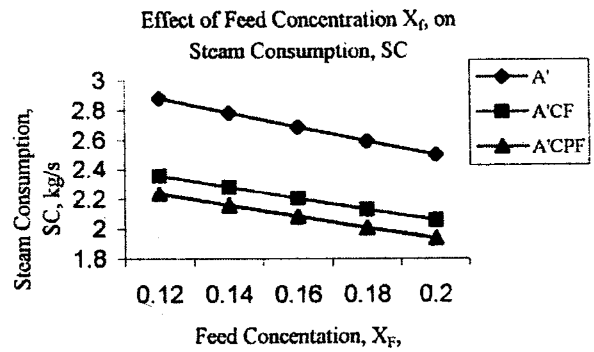

Fig. 10

of area demand is noted, if T_n is used as a parameter .

From the analysis of these alternations in design, the following observations are noted.

a. Reduction of SC, on an average of 17.78% for shifting from $A' \rightarrow A'_{CF}$, 5.5% from $A'_{CF} \rightarrow A'_{CPF}$ & 22.31% from $A' \rightarrow A'_{CPF}$ are achieved.

b. Enhancement of SE for the same change, is found of the order of 21.59% from the change of design $A' \rightarrow A'_{CF}$, 5.82 from $A'_{CF} \rightarrow A'_{CPF}$ & 28.67% from $A' \rightarrow A'_{CPF}$ are obtained.

c. Area demand, A increases by 9.04% from $A' \rightarrow A'_{CF}$, 1.5% from $A'_{CF} \rightarrow A'_{CPF}$ and 10.74% from $A' \rightarrow A'_{CPF}$.

It is very much clear that flashing of condensate and product flash can bring large amount of steam saving vis a vis a profit to the industry.

Effect or Feed Concentration

Feed concentration of black liquor is also an important parameter for evaporation operation in paper industry.It changes with the change of raw material, pulping conditions and the pulping processes. The concentration normally lies between 14-18% for wood/ bamboo based raw material. The effects of this parameter on SE, SC and A are shown in fig. 10-12. It is reflected that with the increase of feed concentration all the three parameters, namely, SC, SE and A decrease.

With the increase of feed concentration, X_f, all the three parameters SC, SE & A decrease. The steam consumption, SC decreases to a value, on an average of 18.05% from $A' \rightarrow A'_{CF}$, 5.37% from $A'_{CF} \rightarrow A'_{CPF}$ and

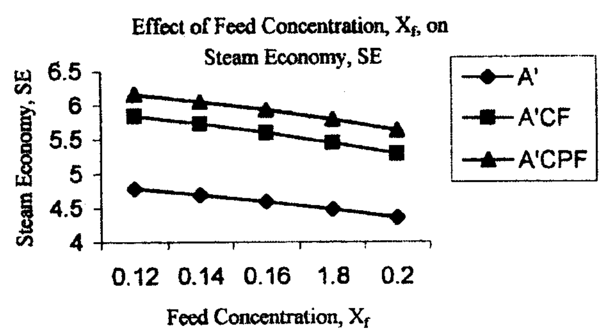

Fig. 11

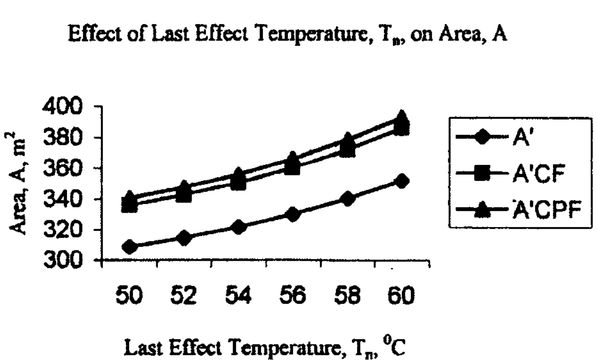

Fig. 9

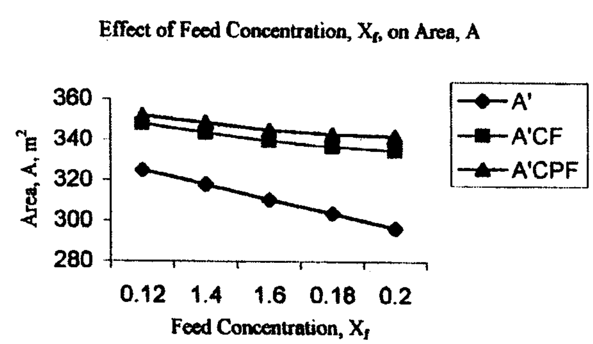

Fig. 12

22.46% from $A' \to A'_{CPF}$. Around 22.00 % increase of SE from $A' \to A'_{CF}$, 5.67% $A'_{CF} \to A'_{CPF}$ & 28.92% from $A' \to A'_{CPF}$ are also obtained. This indicates an enormous quantum of steam saving. On the other hands, extra heating surface requirements increase by 8.2% form $A' \to A'_{CF}$, 1.3% from $A'_{CF} \to A'_{CPF}$, & 9.68% form $A' \to A'_{CPF}$. It is evident that the increases in area demand are very small compared to the gain in economy due, to reduction in steam consumption.

For better clarity, a few noteworthy results are shown in the following table.

evaporator system of any other configuration to estimate steam economy, SE, steam consumption, SC and area A.

Nomenclature

A Area of evaporator body, heat transfer area, m^2

a Fractional heating area of the effect denoted by a_j = A/(50.F), sm^2/kg

BPR Boiling point rise, °C or Kelvin

C Condensate from the steam chest

C_p Specific heat of liquor, kJ/kg °C

Table 3

O.P.	Mixed feed Sequence					
	No flash, A'	With condensate flash, A'_{CF}	With condensate & product flash, A'_{CPF}	Diff. $A' \to A'_{CF}$ %	Diff. $A'_{CF} \to A'_{CPF}$ %	Diff. $A' \to A'_{CPF}$ %
Steam Economy, SE, kg/s	4.6350	5.6520	5.894	21.94	4.28	27.16
Steam Consumption, SC, kg/s	2.7254	2.2344	2.1424	18.01	4.11	21.39
Area, A, m^2	313.418	341.238	345.013	8.8	1.10	10.08

CONCLUSION

On closer scrutiny of the results, the following inferences can be made :

Around 21.94 % steam saving is possible at normal working conditions with condensate flash in comparison to those from a system without flash. Saving of Steam to an extent 27.16 % is possible with flashing of condensate plus product flash.

It is cleat that in the mixed feed sequence (when feed enters in fifth effect) with condensate flash A'_{CF} is always better than A' and A'_{CPF} (with condensate and product flash) is better than A'_{CF}.

The developed mathematical model can be used to many chemical and process industries after subsequent changes which use mixed feed multiple effect

F Liquor feed rate, kg/s

f Functions obtained by models equations

g A function defined by $g_j = f_j/F.\lambda o$

H Enthalpy of the vapour, kJ/kg

h Enthalpy of liquor. kJ/kg

L Liquor flow rate, kg/s

M Flash vapour, kg/s

n Number of effects

Q Rate of heat transfer across the tube from the steam/ water vapour to the liquor, w

T Saturation temperature of water at pressure P, °C.

U overall heat transfer coefficient, W/m^2k

V Vapour flow rate from the effect, kg/s

x Mass fraction of solute in the liquor

l,m,u, v are scaled liquor flow rate, flash vapour rate, temperature flow rate and vapour flow rate defined by $l_i=L_i/F$, $m_i=M_i/F$, $u_i=T_i/T_0$ and $V_i=V_i/F$, respectively.

Subscripts and Greek Letters

f Feed concentration

p Final or output poduct

s Steam, saturation

v Vapour

λ A Latent heat of vaporization, kJ/kg

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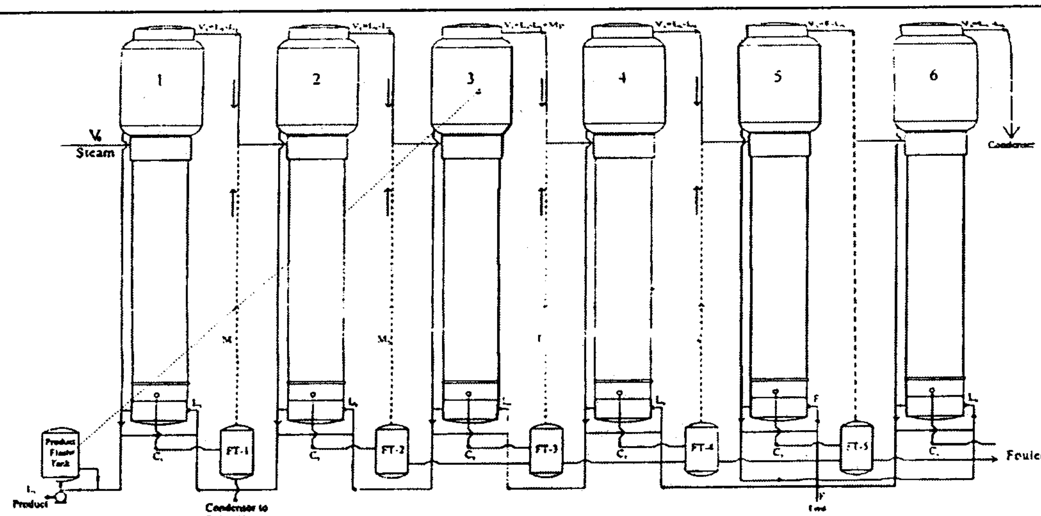

Sextuple Effect Evaporator System with Mixed Feed and with Condensate Plus Product Flash Tank (5→6→4→3→2→1)