Optimization of Multi-Stage Bleach Plant

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ABSTRACT

In this present investigation a pragmatic strategy for optimizing a multi-stage a bleach plant has been developed. To fulfil the objective a detailed optimization scheme has been designed which is then simulated for a typical bleach plant sequence CEDED as an example. Non linear models suggested by earlier workers, especially from the FOXBORO Company, USA, for various bleaching stages of this sequence, linearization techniques applied, the operating constraints specified and the linear programming adopted are examined, moderated and presented in more lucid forms, understandable to a process engineer and a mathematician. The models are then solved with incorporating various equations of constraints through the linear programming (LP) with the help of MATHCAD AND LINDO SOFTWARE PACKAGE. As usual the control set points based on the models are optimized using annual operating cost as decision variables. The total annual cost as objective function has been formulated and simulated with various values of operating conditons such as consistency, temperature, time of reaction, chemicals consumed and final pH within their respective ranges normally followed by industry.

The extended optimization scheme proposed are found to be most appropriate and efficient one that can be used to any bleach plant and can save enormous expenditure in bleaching due to saving in chemicals and steam. However as the costs of bleach chemicals vary from plant to plant and time to time, the illustrated outline is a conservative one. Every plant operator should exercise the above strategy from time to time to save energy and chemicals with a view to reap financial benefits in their mill. The procedure can further be employed to other bleaching sequences, typically, CEH and CEHH etc. which are overwhelmingly major bleaching sequences in Indian Paper Mills.

INTRODUCTION

Ever increasing costs of chemicals, energy and other inputs and strict legislation of the pollution control authorities for treatment of the generated effluents due to the chemical reaction between the reactant and pulp, has forced the paper Industry to optimize various processes and operations to be effective and work in a most economical way.

Bleach plant is an important subsystem of a Pulp and Paper Mill. The bleaching process consist of three steps namely bleaching, particle removal and brightening.

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In a bleach plant bleaching of pulp usually takes place through several unit processes and operations. The processes are connected with many chemical reactions depending upon the nature of oxidizing (oxidative bleaching) or reducing (reductive bleaching) or enzymatic agents applied to the pulp. The chemical reactions or the biochemical reactions (in the later case) are very complex. The reactions also are of many types (principal and side reactions). The operations involved toperform the task of bleaching needs single or multistage bleaching sequences, though the later is almost invariably used throughout the world except in very few cases of pulp like those of non wood base pulp depending upon the brightness required. The operation is normally accomplished in a series of equipments like mixer, retention towers, washers and a couple of pumps for pulp slurry transportation. The operational parameters are usually consistency of residual chemicals and the final pH. Most difficult point is that chemical reaction in each stage demands different conditions of the above parameters. The chemical applied on a pulp is normally determined by an empirical equation using a kappa no. or a permanganate no., and a multiplying factor arbitrarily defined. This makes the system modeling an extremely difficult task.

An attempt to reduce the cost of bleach plant can enhance significantly the overall economy in a pulp and paper mill.

Bleach plant optimization is one of the steps which can help reduce the consumption of these inputs to minimize pollution load generation in paper mills and to cut down costs.

To perform bleach plant optimization one has to select an objective function for minimizing the total cost of operation of a bleach plant. The cost of a bleach plant depends primarily on the following points.

- Cost of bleaching chemicals which is a function of many variables including wood species, kappa no. entering bleach plant, bleach sequence, retention time in towers and the operating temperature in the towers and the conditions of the inter-stage bleach washing and the mixing efficiency in the mixer.

- The cost of bleaching is also a function of the degree of removal of the previous stage bye-products as well as pH adjustment in the succeeding stage, besides the cost of energy required in pumping, mixing, inter stage washing plus the steam requirement for adjusting the temperature.

The objective function mainly consists of cost of chemicals, cost of steam, cost of electrical power plus the repair and maintenance expenses for large number of equipments employed in any bleach plant under consideration. In many cases, the major expenditure for equipment and tankage are difficult to justify because of uncertainty of repair and replacement of bleach plant components due to wear and tear, and corrosion. It is also difficult to estimate the product formed out of the reaction which is reagent and sequence specific.

Practically limited attempts have been made for modelling or optimization of bleach plant with a view to improve efficiency vis a vis to reap economic benefits.

Shackford (1) had attemped to present an optimum design and operation to achieve the best results possible for a bleach washer, Unfortunately this is confined to a segment of a bleach plant, not bleach plant as a whole. Freedman (2) of FOXBORO Company, USA, has developed optimization in a true sense for a specific sequence CEDED bleach plant using a non linear model which is linerized by Taylor series expansion and the coefficients are determined from the plant data reported by Moor and Haner (3). Substantial cost benefit of nearly 14.5% due to saving in chemicals and steam has been indicated. No other works are reported to-date to the author's knowledge.

In this present investigation a bleach plant of identical sequence CEDED has been assumed just to examine the applicability of the models presented by them at the first stage as a part of the investigation and then extending forwider range of data of the variables. A typical diagram of CEDED sequence is shown in Fig. 1. The functions of each stage is very much clear from the diagram. Further enlarging of the stage (like CEHDED, CEDEDH/P etc.) has not been attempted as it is well known that longer the sequence degree of difficulty is more for modelling, optimization and control of the bleach plant.

DEVELOPMENT OF MATHEMATICAL MODEL

Modelling is a mathematical description of physical system (process/operation) or subsystem which are simple, coherent, less time consuming and made with most pertinent parameters influencing the system, open to parametric treatment, easily solvable and controllable. Model building provides some logical and systematic approach for understanding the design and analysis of the problem and also undertakes

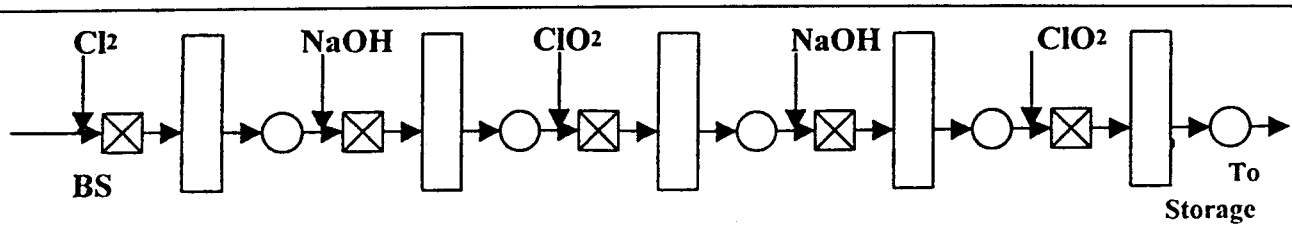

Fig.1 A five-stage bleach plant consisting of chlorination, alkaline extraction, chlorine dioxide,alkaline extraction, and chlorine dioxide stages.

optimization through economic descriptions and explanations of the operations of the system they represent.

Dynamic programming has not been applied so far in the optimization of the bleach plant though it is a multistage process possibly, because the plant itself is discrete in each and individual stage unlike multistage evaporator and becomes terminated with an inclusion of a washer or a mixer.

The software developed by FOXBORO is shrouded with secrecy and no concrete conclusions can be made out of the published information. The model based on LP (2) has the following limitations:

- It gives optimization on a fix set of operating variables it does not apply to an wide range of the operating conditions and the type of the pulp produced from the various species.
- The parameter of the model are determined from the plant data presented by Moor and Haner (3).
- Constraints for parameters are given in mathematical minimax notation only as an example for temperature and chemicals used only in CE stage. Constraints for other stages, D or ED are not given.

More realistic approach is presented here. The steps followed are:

- Model Building through kinetic data for single and combined stage.
- Development of statistical multi-variate regression equation
- Linearizing the models
- Determining of Coefficients
- Selecting the Constraints
- Formulating the objective function
- Solving through linear programming

These are discussed hereunder

MODELLING OF BLEACH PLANT

For a multistage bleach plant like CEDED, One can develop a separate model for each stage but each does not have a large effect on the target variables (brightness, strength etc.) Therefore, it is prudent to assume one model to show combined effect of two stages. For the first two stages (CE), equations (1) and (2) are used. For each succeeding stages (D) and (ED), two general equations (3) and (4) are employed. These equations describe the effects on kappa number, residual chemicals, brightness of changing chemical, temperature, retention time, and brown stock kappa number. The nonlinear aspects of models in some stages are also considered. The parameteric influence on the desired parameter is usually presented by a polynomial of any order to take care of nonlinearity. The coefficients are normally evaluated through least square techniques for multiple regression analysis. Using the above statistical method, **the regression equations of Kappa Number (w_1), residual chemicals (r), brightness (B) and brightness for nth stage (B_n), residual chemicals for nth stage (r_n), can be written in nonlinear- orthogonal form, as follows:**

$$w_1=w_{10}+a_1(w_2-w_{20})+a_2(w_2-w_{20})^2+b_1(x-x_0)+b_2(x-x_0)^2+c_1(y-y_0)+c_2(y-y_0)^2-s_0)+d_2(s-s_0)^2 \quad (1)$$

$$r=r_0+a_3(w_2-w_{20})+a_4(w_2-w_{20})^2+b_3(x-x_0)+b_4(x-x_0)^2+c_3(y-y_0)+c_4(y-y_0)^2+d_3(s-s_0)+d_4(s-s_0)^2 \quad (2)$$

$$B_n=B_{n0}+e_1\Delta w_{10}+e_2(\Delta w_{10})^2+f_1\Delta x_0+f_2(\Delta x_0)^2+g_1\Delta y_0+g_2(\Delta y_0)^2+h_1(\Delta s_0)+h_2(\Delta s_0)^2+j_1\Delta u_0+j_2(\Delta u_0)^2 \quad (3)$$

$$r_n=r_{n0}+e_3\Delta w_{10}+e_4(\Delta w_{10})^2+f_3\Delta x_0+f_4(\Delta x_0)^2+g_3\Delta y_0+g_4(\Delta y_0)^2+h_3(\Delta s_0)+h_4(\Delta s_0)^2+j_1\Delta u_0+j_2(\Delta u_0)^2 \quad (4)$$

ON THE LINEARIZATION OF THE MODELS

It is imperative to linerize a non linear model by a series expansion either Maclaurin's or Taylor's series expansion for further analysis. In this work the nonlinear model equations are linearized by a general Taylor's series expansion and neglecting the higher order terms, the following equations yield:

$$w_1=w_{10}+a\Delta w_{20}+b\Delta x+c\Delta y+d\Delta s \quad (5)$$

$$r=r_0+a\Delta w_{20}+b\Delta x+c\Delta y+d\Delta s \quad (6)$$

$$B_n=B_{n0}+e\Delta w_{10}+f\Delta x+g\Delta y+h\Delta s+j\Delta u \quad (7)$$

$$r_n=r_{n0}+e\Delta w_{10}+f\Delta x+g\Delta y+h\Delta s+j\Delta u \quad (8)$$

where $\Delta x=x-x_0$

If a variable does not have a large effect, the variable would be dropped from the model for that particular stage.

For each plant there is a range of parameters in which the model is valid. For different set of ranges of parameters one has to calculate new coefficients. In equation (1), extracted kappa number is a function is of temperature, chemical used, and retention time in chlorination stage. By drawing a curve (it may or may not be nonlinear) and projecting the straight line on this operating curve from reference point to the current operating point gives the range of parameters within which the linearization is valid.

HOW TO DETERMINE THE COEFFICIENTS

As already indicated the bleaching operation is an extremely complicated process which involves a number of variables. It is very difficult to correlate various input to output parameters for any bleach plant, even for a single stage. One can however optimize the bleaching operation by defining the cost function and constraints for one's own brightness, kappa number, and residual chemical specifications. Some of the cost parameters are strongly dependent on many input parameters. the output parameter depends on pulp type, temperature, consistency, pH, time and initial kappa number. Though large body of data and graphics are available regarding optimization of bleaching, no mathematical treatments are given except the one reported above (2). However, the solution technique has not been shown anywhere. In this present investigation, functional relationship for each variable has been developed by Multiple Regression analysis using the software named "MATHCAD". The data reported in Singh (11) are employed for regression equations Eq. 11 and 13 and data of Histed and Canovas (10) are also used for regression equation, Eq. 9 to determine the coefficients. The coefficients given by Freedman (2) and those calculated in the present investigation are depicted in Table-1.

It is worth mentioning that all the coefficients presented by Freedman and computed by this present software are found to be different due to different conditions of experimental data.

SELECTING THE OPERATING CONSTRAINTS

It is well known that a model can be valid or accurate if it contains all the objectives, constraints, and decision variables relevant to the problem or actually part of the problem. The operating conditions are used as contraints because each plant has a limit for a operating variable. For example, each sequence has a limit of chemicals used, temperature, retention time etc. in a particular stage. These limits are put in the form

Table-1 Coefficients of equation 9, 11 and 13

i	a_i		b_i		c_i	
	Ref. (2)	Present	Ref. (2)	Present	Ref (2)	Present
0	33.5	7.67	80.0	1.977	7.0	1.633
1	0.5	.072	-25.0	-1.118	0.2	0.276
2	-0.15	-0.068	2.5	11.86	0.3	9.797
3	-0.1	-0.014	0.3	0.054	0.3	0.038
4	-0.03	-0.011	0.01	0.019	0.01	.003296

of mathematical in euquations. These are then used as constraints for the LP. If we use large amount of chemicals, high temperature it will cause the accelerated color reversion or poor strength, higher pollution load and more energy consumption. Mimimum time for completion of the reaction to get desired value of brightness is also needed for each stage. Therefore, it is important to use the set of constraints (operating variables) properly. These constraints can be examplified for stages (combined or single as the case may be) as:

$x_1 \geq 60$ kg/tonne of pulp	$s_1 \geq 120$ min	$y_1 \geq 40^0C$
$x_2 \geq 8$ kg/tonne of pulp	$s_2 \geq 180$ min	$y_2 \geq 60^0C$
$x_3 \geq 6$ kg/tonne of pulp	$s_3 \geq 120$ min	$y_3 \geq 70^0C$

FORMULATION OF AN OBJECTIVE FUNCTION

For a multistage bleach plant the objectives are usually multi choices like maximum brightness, minimum pollution load generation (BOD, COD, AOX, SS, TDS, TOCI etc.), minimum color reversion (post color number), maximum throughtput, minimum strength loss and minimum cost etc.

Here like previous worker, the last one is taken as the objective function considered to be the linear function of decision variables (the major cost i.e. chemical cost, steam cost etc.) of the bleach plant, which is to be minimized. The objective function thus can be formulated for a specific target variable (brightness, strength) as follows:

$$Z=C_0+C_1x_1+C_2x_3+C_3x_5+C_4y_1+C_5y_3+C_6y_5$$

The above objective function can be subjected to LP to minimize the cost to get optimal values of the parameters.

RUN THE LINEAR PROGRAMME

The process of solving an LP requires a large number of calculations and is therefore best performed by a computer program. In the present work the computer software package known as LINDO can be used. The main purpose of LINOD is to quickly input an LP formulation, solve it, assess the correctness or appropriateness of the formulation based on the solution, and then quickly make minor modifications to the formulation and repeat the process. To solve the LP problem LINDO requires operating data of various stages of CEDED bleach plant. The operational data presented by Freedman are used to check their validity. It agrees excellently without any noticeable error. In this work the operating data due to Sklarewitz and Parker (4) given in Table-2 are used as initial conditions.

OPERATING DATA FOR SIMULATION

It is very difficult to estimate the cost of individual chemicals used for industrial usage as it varies not only with the purity of product but also with the market trends for escalation of cost indices.

However it is fairly accurate to assume the constant ratio of cost of bleaching chemicals which are available elsewhere. The ratio of the costs of the chemicals reported by various investigators is depicted in Table-3.

TABLE-2 Typical and Calculated Operating Data for a CEDED Bleachery (4)

	Temperature °C		Consistency %		Bleach chemical charge, %		Time		Final pH	
Stage	Typical	Calculated	Typical	Calculated	Typical	Calculated	Typical	Calculated	Typical	Calculated
C	20	20	3-4	3.78	5-7	5.70	40 min	40 min	<3	2.6
E	70	70	12-14	11.9	3-4	1.19	2 hr	2 hr	11	11.99
D	70	70	12-14	13.9	0.6-1.0	0.93	3 hr	3 hr	3.5-4	4.8
E	70	70	12-14	14.0	0.5	-	2 hr	2 hr	11	9.3
D	70	70	12-14	14.0	0.3-0.5	0.85	3 hr	3 hr	5-6	8.8

TABLE-3 Ratio of the cost of chemicals

Reference	Chemicals	Ratio
(2)	Cl_2: NaOH:NaOCl:ClO_2:	1:1:2.15:5.72
(9)	Cl_2:NaOH	1:1.3
(12, 17)	O_2:H_2SO_4:NaOH:$MgSO_4$:ClO_2 (as active Cl_2):H_2O_2:O_3:Chelating agents	1: 1.14:3.14:5.29:6.14: 12.57:17.14:21.42
(13)	O_2:NaOH:Cl_2:ClO_2	1:1.375:1.375:2.5
(14)	O_2:$MgSO_4$:NaOH:Cl_2:ClO_2 (as active Cl_2):	1:1.2:1.4:1.25:2.5
(15)	O_2:HOCl:ClO_2:H_2O_2	1:4:9:12
(16)	Silicate 41°Be:SO_2:NaOH:Na_5DTPA: $Na_2S_2O_{24}$:H_2O_2	1:1.57:1.72:7.29:8.86:9.14

In this present work, however the ratio of cost of chemicals (Cl_2:NaOH:NaOCl:ClO_2) can be taken as 1:1.3:2.14:5.72. Using this ratio it is easy to find out the cost of other chemicals if one of them is known. Using an average ratio of costs of bleaching chemicals (2, 9) given in the table and taking the cost of chlorine from Jim Haynes et al. (9) other chemical costs are calculated. This is shown in Table-4.

SAMPLE CALCULATION

In this investigation efforts are being made to make this model easy to understand. In order to optimize the conditions various equations of constraints are moderated. Coefficients of regression equations are determined tosolve the LP. For this purpose MATHCAD AND LINDO SOFTWARE PACKAGE are used. The objective function is a linear function of total annual cost. The influences of various parameters are used for simulating the models.

An example is given here to describe the optimization procedure for CEDED bleach plant. The derived models for first two stages are:

$$w_1=a_0+a_1\Delta w_{20}+a_2\Delta x_1+a_3\Delta y_1+a_4\Delta s_1 \quad (9)$$

$$r=r_0+a_5\Delta w_{20}+a_6\Delta x_1+a_7\Delta y_1+a_8\Delta s_1 \quad (10)$$

The model for third stage (chlorine dioxide) is as follows.

$$B_3=B_{30}+b_1\Delta w_1+b_2\Delta x_3+b_3\Delta y_3+b_4\Delta s_3 \quad (11)$$

$$r_3=r_{30}+b_5\Delta w_1+b_6\Delta x_3+b_7\Delta y_3+b_8\Delta s_3 \quad (12)$$

The model for fourth and fifth stage (extraction, chlorine dioxide) is

$$B_5=c_0+c_1\Delta b_3+c_2\Delta x_5+c_3\Delta y_5+c_4\Delta s_5 \quad (13)$$

$$r_5=r_{50}+c_5\Delta b_3+c_6\Delta x_5+c_7\Delta y_5+c_8\Delta s_5 \quad (14)$$

TABLE-4 Cost of bleaching chemicals

Chemical used	Price per chemical, Rs/kg	Steam cost (°GE88-90), Rs/T
Chlorine	6.48	500
Sodium Hydroxide	8.33	
Sodium Hypochlorite	13.88	
Chlorine Dioxide	37.03	

Table above unit costs given in Table 4 are typically used to formulate the objective function.

It should be noted that the pH dependence is not included in this model.

In general, the model can be expressed as:

$$\begin{pmatrix} \Delta B_1 \\ \Delta B_2 \\ \Delta B_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \end{pmatrix} \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \\ \Delta x_5 \end{pmatrix}$$

where: ΔB_1= extracted kappa number, ΔB_2= brightness, ΔB_3=residual chemical, X_1= previous stage kappa number or brightness, X_2=chemical addition, X_3= tower temperature, X_4=residence time, X_5=pH, a_{ij}=coefficient.

To make the calculation easy the equations for extracted kappa number from 1st stage (Eq. 9), brightness from stage 3rd and 5th (Eq. 11 and 13) of bleach plant model are used. Using the coefficient from Table-1 the model equations 9, 11 and 13 can be expressed as

$$w_1=7.672+0.07*w_2+0.068*x_1-0.014*y_1-0.011*s_1 \quad (15)$$

$$B_3=1.977-1.118*w_1+11.86*x_3+0.54*y_3+0.019*s_3 \quad (16)$$

$$B_5=1.633-0.276*B_3+9.797*.x_5+0.038*y_3+0.03296*s_5 \quad (17)$$

In stages IVth and Vth (D & ED stages) cost of relative amount of chemicals used is considered. Using the coefficient for the conversion of stage temperature to steam costs and from Table-4 using prices for bleach chemical the objective function can be expressed as:

$$Z= - 148.14+6.48*x_1+37.03*x_3+17.89*x_5+10.08*y_1+1.08*y_3+2.16*y_5$$

The objective function when subjected to the constraints (Eq. 15, 16 and 17) gives the optimum conditions, presented in Table-5.

For final stage brightness 89.89°GE, using the current operating conditions given in Table 2 and the optimum conditions in Table-5, the total cost involved and the benefits accrued can be written as follows:

Table-6 shows a decrease in bleaching cost of Rs. 300/Tonne after using the optimized conditions. Therefore, the model developed by Freedman (2) is found to be most appropriate and efficient one that can be used to any bleach plant and can save chemicals and energy. As a result enormous expenditure in bleaching can be avoided. The procedure can be further employed to other bleaching sequences, typically, CEH and CEHH etc. which are overwhelmingly major bleaching

TABLE-5 Results (optimum conditions) for final stage brightness 89.89 GE

Stage	Chemical Addition kg/tonne		Temperature, °C		Brightness or Kappa number	
	Ref. (2)	present	Ref. (2)	Present	Ref. (2)	Present
CE	71.42	60	37.77	40	4.46	4.40
D	2.23	6	60	60	75.0	74.81
ED	15.63	6.69	80.55	70	88.0	89.89

TABLE-6 Comparison of Total before and after optimization

Total Cost, Rs per Tonne of Pulp		Total savings	
Before optimization	After optimization	Percentage	Rs. per Tonne
1502 (35.76$)	1200 (28.57$)	20.0	300

sequences in Indian Paper Mills.

CONCLUSIONS

- A modified form of non-linear system model of a multi-stage bleach plant has been developed based on the guide lines of previous workers for optimization For mere convenience CEDED has been taken as an example. The optimization scheme of Freedman has been restrucured, linearization methodology has been simplified and linear programming have been solved through **MATHCAD AND LINDO SOFTWARE PACKAGE.** The control set points based on the models are optimized using annual operating cost as decision variables. The total annual cost as objective function has been formulated. The various parameteric influences are used for simulating the models. These are then tested when subjected to various values of operating conditions such as consistency, temperature, time of reaction, chemicals consumed and final pH within their respective ranges nromally followed by industry.

- Optimization scheme can boost the significant economy by saving chemicals and steam. Preliminary computation on the assumed costs of inputs at the present market rates estimates the cost. Saving on the order of Rs. 300 per tonne of bleached pulp can be anticipated which is substantial if considered annually. However as the costs of bleach chemicals vary from plant to plant and time to time, the illustrated outline is a conservative one. Every plant operator should exercise the above strategy from time to time to save energy and chemicals with a view to reap financial benefits in their mill.

- The procedure can be further employed to other bleaching sequences, typically, CEH and CEHH, CE_0H etc. with or without re-inforcement by peroxide addition and chlorine-di-oxide substitution which are overwhelmingly major bleaching sequences in Indian Paper Mills.

- The model has become more easy to understand to both mathematician and process engineer working in pulp and paper industry for optimizing the complicated bleach plant.

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NOMENCLATURE

B_n = brightness from n^{th} stage, % GE

C_0 = base cost of bleaching

C_1 = price of chlorine in stage 1 and caustic in stage 2, Rs/kg

C_2 = price of chlorine dioxide in stage 3, Rs/kg

C_3 = price of chlorine dioxide in stage 5 and caustic in stage 4, Rs/kg

C_4 = price of steam in stages 1 and 2 times the conversion of steam use to temperature changes, Rs/^{0}C

C_5 = price of steam in stage 3 times the conversion of steam use to temperature changes in stage 3, Rs/^{0}C

C_6 = price of steam in stage 4 and 5 times the conversion of steam use to temperature changes, Rs/^{0}C

Z = Objective function, Rs/tonne

a_i, b_i, c_i, d_i, e_i, f_i, g_i, h_i, j_i = coefficients

r_n = chemical residual in n^{th} stage, kg/tonne of pulp

s_1 = residence time in 1st and IInd stage, min

s_3 = residence time in IIIrd stage, min

s_1 = residence time in VIth and V^{th} stage, min

u = pH of stage n

x_1 = chemical used in1^{st} and IInd stage, kg/tonne of pulp

x_3 = chemical usage in stages 3, kg/tonne of pulp

x_5 = chemical usage in stages 4 and 5, kg/tonne of pulp

y_1 = 1st stage temperature, ^{0}C

y_3 = IIIrd stage temperature, ^{0}C

y_5 = V^{th} stage temperature, ^{0}C

w_1 = extracted kappa number

subscript, 0, refers to a basic fixed reference point.